Optimal Compensation with Induced Moral Hazard in Investment*

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Abstract. We extend the principal-agent framework in the sense that the principal can neither verify the agent's effort choice nor his investment strategy. In this setting, we provide a rationale for compensating the manager with equity based pay in a manner closely related to a call option. Moreover, contrary to the common belief, we show that maximizing the "incentives" by standard measures used in the finance literature induces the manager to an inoptimal investment and effort choice. Finally, benchmarking the manager's compensation by market additional information turns out to be far from straightforward.

Keywords. Optimal compensation, executive stock option, investment, principal agent model.

JEL classification numbers. G31, G34.

1 Introduction

An important element in Corporate Governance is the set of mechanisms which serve to influence management decisions when ownership and control are separate. One important mechanism is the compensation of managers. As Corporate Governance in general, stock and option compensation are hotly debated topics. Naturally, the proliferation and especially the level of equity incentives have contributed to the interest. Hall and Liebman (1998) report that in 1994 approximately 90% of US top management held options as part of their portfolios and Hall and Murphy (2002) find that 94% of the S&P 500 companies issued options to management in 1999. The value of options granted and equity instruments held is significant. For S&P 500 companies Hall and Murphy (2002) find that in 1999 the grant value of options awarded constituted 47% of annual top management compensation and the median market value of top management equity instrument holdings was $31 million – the latter applies to the industrial companies in S&P 500.

As part of the governance structure, the declared purpose of most option grants and stock appreciation rights awarded is to attract, retain, and motivate management as well as lower level employees. Given there are other means of achieving these goals – bonus schemes, lump sum payments, and regular salary – a natural question is whether equity based compensation is efficient. As efficiency concerns whether a given purpose is achieved cost effectively, it seems efficiency of equity based incentives can be analyzed in two parts. One part concerning the cost and another part concerning the incentive effects of equity based programs. Though seemingly straightforward, the cost of an option grant is hard to determine. Options granted can be European or American and can have all kinds of vesting periods. As the recipient is risk-averse and the options granted are usually non-tradable and non-replicable, the recipient might pursue an otherwise inoptimal exercise strategy. Furthermore, the market value – given the recipient's exercise strategy – and the value to the recipient as measured by the certainty equivalent of the option grant can differ significantly. This has given rise to the perception that option contracts are inefficient because the owners pay for more than they get.[1] This position could have some merit as the difference between cost and certainty equivalent – the risk premium – is part of agency costs.[2]

It turns out, however, that separation of the problem into a cost-benefit framework is problematic. Analyzing costs is difficult, but estimating benefits turns out to be even more problem-

[1] Oddly, the opposite conclusion has not been reported even though the certainty equivalent can be higher than the market value.

[2] Agency costs consist of two parts. One part is the efficiency loss caused by the change in production decisions – relative to the situation where these choices are contractible. The other part is the efficiency loss caused by inefficient risk-sharing inherent in resolving the agency problem.

atic. In the literature, a common proxy for the benefit of option incentives is the sensitivity of the manager's (option) portfolio with respect to the current share price. Regardless of whether the sensitivity is measured as the hedge ratio or the derivative of the certainty equivalent with respect to current share price, the underlying assumption is that the manager can affect only the first moment in the distribution of future share price. If this assumption does not hold, the manager might have an incentive to reduce market value. Lambert, Larcker and Verrechia (1991) demonstrate that a risk averse manager holding a call option can have an incentive to reduce variance despite the fact that this reduces the market value of the option (and of the company). Even more problematic is the fact that incentive effects of option contracts are evaluated without explicit modeling of how option-like contracts can arise endogenously. It is an open question what conclusions can reasonably be drawn if the incentive problem under consideration is exogenously assumed to be most efficiently resolved employing an option contract.

The incentive effect of an option grant gives rise to another problem which is easily missed if the efficiency consideration is separated into costs and benefits. The problem is that incentive effects and pricing are interdependent. Thus, it is not without problems when Hall and Murphy (2002) try to maximize the incentive effects as measured by the sensitivity of the manager's certainty equivalent to current share price holding both the cost and manager certainty equivalent constant. The market value of the company, the firm's cost, and the manager's certainty equivalent are all affected by incentives, however, it is unclear whether this is taken into consideration. We demonstrate that maximizing incentives via the procedure suggested by Hall and Murphy (2002) can have devastating effects. If the agent can increase variance without bounds, and the agent faces an option contract with sufficiently high incentives, the risk is he will overinvest to increase variance. Hence, our analysis suggests that if it is possible, then it would be advantageous to separate investment and operating decisions, even if it leads to a loss of efficiency in the first-best situation.[3]

Many of the procedures recommended in the practice oriented literature – e.g. Activity Based Costing, Balanced Scorecard and the use of options – have nice aesthetic properties; however, they are rarely derived endogenously. We develop a parsimonious model in which costs and benefits are considered in a unified setup, and in doing so we provide an explanation for convexity in compensation in the sense that contracts are convex-like when incentives for both effort and investments must be provided, while convexity is less convincing when only incentives for effort are needed. Core and Qian (2002), Feltham and Wu (2001), Hemmer, Kim and Verrechia (1999), and Lambert (1986) pursue similar strategies in the sense that effort

[3]We thank A. Arya for pointing this out to us.

either affects variance or the agency problem entails project selection. Our model differs from the beforementioned papers in the sense that we study a setting where the agent makes two decisions. The agent exerts effort, which affects the mean outcome, and the agent undertakes investment, which affects both mean and variance. Both the effort and investment opportunity sets are unbounded and the decision variables can be varied independently. The agent has direct preferences over effort decisions but no direct preferences over investments. This leads to an induced moral hazard problem where the compensation scheme needed to induce effort leads to an induced moral hazard problem regarding investments. Furthermore, our model allows for a study of the agent's risk taking behavior, when the agent is exposed to option contracts.[4]

Our formal demonstration of convexity in compensation proceeds as follows. The key elements of our model are described in section 2. Section 3 presents a benchmark case where investments are either observable or controlled by the principal. Section 4 contains the analysis of the situation where incentives for both effort and investment must be provided, while sections 5 and 6 present an analysis of the problem where the shape of contracts is restricted. Section 7 addresses the question of whether and how additional information should enter the compensation arrangement and we conclude in section 8.

2 Model

We consider a one-period principal-agent model in which an important variation on the usual story is introduced: the agent has available a certain amount of working capital, which the agent invests in two distinct technologies, one risky and one riskless technology. The available working capital is given exogenously and, hence, it is not a decision variable. We initially assume the optimal investment in the risky technology is less than the available working capital and that the principal can observe any borrowing by the agent. That is, the agent cannot invest in excess of the capital already at his disposal.[5]

The principal observes neither effort choice nor the allocation of investment capital between the technologies. And thus, the only contracting variable available to the principal is total payoff or market value resulting from the investment and production decisions made by the agent.

The principal is risk neutral, and the agent is "effort" and risk averse. The action set, A, is convex as is the output set, X. The game begins at time $t = 0$, where a contract (specifying labor supply, working capital supplied, allocation of investments, and compensation function) is agreed upon by the principal and the agent. The agent immediately allocates the available

[4] If there are only two alternative investments, it is less interesting to study the agent's risk taking behavior.

[5] Some of these assumptions are weakened in section 5.

capital, q, between a risky investment, q_p, in a productive technology and a risk-free investment, q_{r_f}, in a financial type asset yielding zero interest. Subsequently the agent supplies productive effort, a, both parties observe final output or market value, $x \in X$, and finally the agent receives remuneration, $s(x)$, from the principal. Thus, we have the time line illustrated in Figure 1.

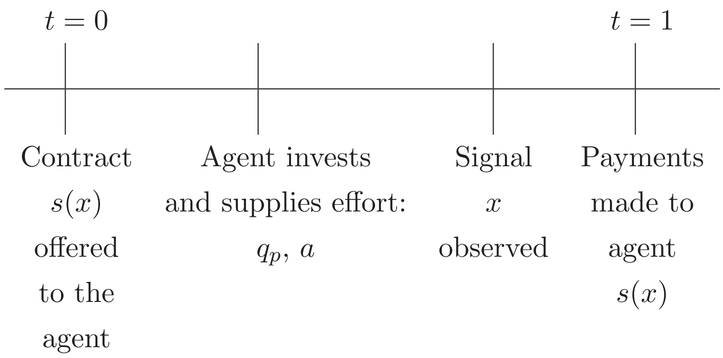


Figure 1: Time line of the model

As noted, the principal is risk neutral and we assume the principal maximizes expected profit, measured as total expected output net of compensation to the agent, whereas the agent's utility depends both on the available consumption, c, and effort, a.[6] That is, the agent's utility is

$$U(c, a).$$

We further assume $U_a(c, a) < 0$, $U_{aa}(c, a) < 0$, $U_c(c, a) > 0$, $U_{cc}(c, a) < 0$, i.e., the agent is risk and effort averse.

The output, x, depends on productive effort and investments as well as on an unobservable state of nature. $g(x\,|a, q_p, q)$ denotes the density over x conditioned on productive effort, a, productive investment, q_p, and investment in the risk-free asset, $q_{r_f} = q - q_p$. We henceforth suppress q and denote the density $g(x\,|a, q_p)$.

The program describing the Pareto optimal contracts can be formulated as follows:

$$\max_{s(x), a, q_p} \int_{-\infty}^{\infty} [x - s(x)] g(x\,|a, q_p)\, dx \qquad \text{[P1]}$$

s.t.

$$\int_{-\infty}^{\infty} U(s(x), a) g(x\,|a, q_p)\, dx \geq R,$$

$$a \in \arg\max_{\hat{a}} \int_{-\infty}^{\infty} U(s(x), \hat{a}) g(x\,|\hat{a}, q_p)\, dx,$$

$$q_p \in \arg\max_{\hat{q}_p \leq q} \int_{-\infty}^{\infty} U(s(x), a) g(x\,|a, \hat{q}_p)\, dx,$$

$$s(x) \geq \underline{\mathrm{s}},$$

[6]Neither the agent's nor the principal's preferences exhibit time preference.

where $s(x) > \underline{s}$ expresses the limited liability of the agent. Assuming the first-order-approach is valid, the principal solves

$$\max_{s(x),a,q_p} \int_{-\infty}^{\infty} [x - s(x)]g(x\,|a, q_p)\,dx \qquad \text{[P2]}$$

s.t.

$$\begin{aligned}
&\int_{-\infty}^{\infty} U(s(x), a)g(x\,|a, q_p)\,dx \geq R,\\
&\int_{-\infty}^{\infty} [U_a(s(x), a) + U(s(x), a)\frac{g_a(x\,|a, q_p)}{g(x\,|a, q_p)}]g(x\,|a, q_p)\,dx = 0,\\
&\int_{-\infty}^{\infty} U(s(x), a)g_{q_p}(x\,|a, q_p)\,dx = 0,\\
&s(x) \geq \underline{s}.
\end{aligned}$$

Proposition 1 *Assuming the incentive constraint is binding on both the moral hazard and the capital allocation constraints, pointwise optimization yields the following first-order-condition with respect to the compensation scheme – where this is interior:*

$$\frac{1}{U_s(s(x), a)} = \lambda + \mu_a \left[\frac{U_{as}(s(x), a)}{U_s(s(x), a)} + \frac{g_a(x\,|a, q_p)}{g(x\,|a, q_p)}\right] + \mu_{q_p}\frac{g_{q_p}(x\,|a, q_p)}{g(x\,|a, q_p)} \tag{1}$$

Proof. See Holmström (1979). ■

Commonly, the assumption regarding the agent's preferences is that

$$U(s(x), a) = -H(a) + J(a)V(s(x)),$$

where either $H(\cdot)$ or $J(\cdot)$ is assumed constant. That is, the agent's preferences are either multiplicatively or additively separable in a and $s(\cdot)$, see e.g. Grossman and Hart (1983). Furthermore, in standard agency models investment problems are usually not modeled explicitly or investments are assumed to be contractible, i.e., [P1] can be solved without the incentive compatibility constraint concerning q_p. Given these assumptions (1) becomes

$$\frac{1}{U_s(s(x), a)} = \lambda + \mu_a \left[J'(a) + \frac{g_a(x\,|a, q_p)}{g(x\,|a, q_p)}\right]. \tag{2}$$

From (2) it is seen that compensation depends on the likelihood ratio g_a/g, which reflects how strongly the output, x, indicates the true distribution from which x is drawn is $g(x\,|a, q_p)$. It follows from (1) and (2) that utility functions and distributional assumptions can be combined to yield convex compensation schemes. For example, if $1/U_s(s(x), a)$ is concave (convex) in $s(\cdot)$ and the right-hand side of (2) is affine in x, then $s(x)$ is convex (concave).[7] Such combinations are utilized by Hemmer, Kim and Verrechia (1999) and Feltham and Wu (2001) to display convexity

[7] Alternatively, if $U(s(x), a) = \ln(s(x)) - H(a)$, then the shape of the compensation scheme is determined solely as a function of the distributional assumptions as $1/U_s(s(x), a) = s(x)$.

in the agent's compensation. Similarly, Lambert (1986) and Core and Quian (2002) show that incentive problems concerning the personally costly identification and analysis of investments and the subsequent investment decision can lead to convexity in the agent's compensation.

Our focus is on induced moral hazard problems. If effort, a, as well as investment, q_p, are contractible, the principal can write a forcing contract and thus the optimal compensation contract is independent of output, i.e., $s(x)$ is constant. As the agent has no direct preferences over investments, the same will hold assuming effort, but not investment, is contractible. The opposite conclusion, i.e., that dealing with the effort incentive problem will lead to efficient investment decisions, does not necessarily hold – regardless of the fact the agent has no direct preferences over investments. As noted, the optimal compensation scheme ignoring the investment but not the effort decision is characterized by (2). However, implementing this compensation scheme might lead the agent to undertake inoptimal investment decisions. In fact, if the compensation schemes characterized in (1) and (2) are not identical almost everywhere we have an induced moral hazard problem concerning the investment decision.

In order to be able to derive tractable results, we assume henceforth that the agent's preferences reflect – in addition to no discounting – constant absolute risk aversion with coefficient r, i.e.,

$$U(s(x), a) = -\exp\{-r(s(x) - C(a))\},$$

where $C(a)$ is personal (monetary) cost of action, which is increasing and weakly convex in action ($C'(a) > 0$ and $C''(a) \geq 0$). Given the agent's effort and production choice, we assume that the expected outcome is described by a production function $f(a, q_p)$ and the risk free investment, where $f(\cdot)$ is strictly increasing and weakly concave.[8] In particular, we assume that

$$x = f(a, q_p) + h(q_p)^{\frac{1}{2}}\tilde{\epsilon} + q - q_p, \tag{3}$$

where $\epsilon \sim N(0, \sigma^2)$, and where $h(q_p)$ is strictly increasing. We can interpret (3) as if the agent's productive investment affects scale – and thus both mean and variance – while productive effort affects only the conditional mean of firm value. These assumptions imply that the density of output is

$$g(x\,|a, q_p) = \frac{1}{h(q_p)^{\frac{1}{2}}\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2}\frac{(x - f(a, q_p) - (q - q_p))^2}{\sigma^2 h(q_p)}\},$$

that

$$\frac{g_a(x\,|a, q_p)}{g(x\,|a, q_p)} = \frac{(x - f(a, q_p) - (q - q_p))f_a(a, q_p)}{\sigma^2 h(q_p)}, \tag{4}$$

[8] That is, $f_a(\cdot)$, $f_{q_p}(\cdot) > 0$, $f_{aa}(\cdot)$, $f_{q_p q_p}(\cdot) \leq 0$, and $f_{aa}(\cdot) f_{q_p q_p}(\cdot) - f_{a q_p}(\cdot)^2 \geq 0$.

and that

$$\begin{aligned}\frac{g_{q_p}(x\,|a,q_p)}{g(x\,|a,q_p)} &= -\frac{1}{2}\frac{h'(q_p)}{h(q_p)} \\ &\quad +\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\frac{1}{\sigma^2 h(q_p)}\left[x-f(a,q_p)-(q-q_p)\right]^2 \\ &\quad +\frac{1}{\sigma^2 h(q_p)}\left\{x-f(a,q_p)-(q-q_p)\right\}\left[f_{q_p}(a,q_p)-1\right] \\ &= -\frac{1}{2}\frac{h'(q_p)}{h(q_p)}+\frac{1}{\sigma^2 h(q_p)}\left\{x-Ex\,|a,q_p\right\}\left[f_{q_p}(a,q_p)-1\right] \\ &\quad +\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\frac{1}{\sigma^2 h(q_p)}\left[x-Ex\,|a,q_p\right]^2.\end{aligned} \tag{5}$$

That is, the likelihood ratio is a strictly convex function of outcome, x.

3 Benchmark case: investment level contractible

As noted, the principal effectively sets the productive investment level, q_p, when the investment is contractible. Inserting (4) into (2), we see that this implies that the first-order condition with respect to the compensation scheme (where this is interior) is

$$\frac{1}{r\exp\{-r(s(x)-C(a))\}} = \lambda+\mu_a[rC'(a)+\frac{(x-f(a,q_p)-(q-q_p))f_a(a,q_p)}{\sigma^2 h(q_p)}] \tag{6}$$

or equivalently

$$s(x)=\max\left\{\underline{s},\frac{1}{r}\ln(r)+\frac{1}{r}\ln\left[\lambda+\mu_a[rC'(a)+\frac{(x-Ex\,|a,q_p)f_a(a,q_p)}{\sigma^2 h(q_p)}]\right]+C(a)\right\}. \tag{7}$$

$s(x)$ is locally – where the compensation scheme is interior – an increasing and concave function (of x).[9] This leads to two observations. Firstly, as a high outcome indicates more strongly the agent did behave, a higher output is considered good news. Secondly, compensation is largely concave in x and local convexity is present only due to the limited liability constraint, $s(x) \geq \underline{s}$.[10] For low level employees the limited liability constraint might be of first order importance; however, we doubt this is the case for executives in large companies. Empirically it seems most compensation packages consist of a relatively high base with option incentives on top. Hence, we do not feel convexity induced by a limited liability constraint is a compelling case for the seeming convexity in compensation contracts. Instead we will study incentives in

[9]Without the limited liability constraint an optimal solution might not exist. According to Mirrlees (1999) it is possible to get arbitrarily close to first-best – the Mirrlees problem. Being casual the problem is the right-hand side of (6) is affine in x. It follows μ_a cannot be positive, since if it is, values of x exist for which the right hand side of (6) and thus marginal utility becomes negative.

[10]We are being casual here; the function is neither convex nor concave.

a setting where the presence of induced moral hazard problems necessitates convexity in the compensation contract.

4 Investment level is unobservable

Given the agent is offered a contract like (7), then – if given the opportunity – the agent has an incentive to reallocate investments and, thus, we have an induced moral hazard problem (regarding investments). When the allocation of capital is unobservable, the first-order approach yields – insert (4) and (5) into (1) –

$$\begin{aligned}\frac{1}{r\exp\{-r(s(x)-C(a))\}} &= \lambda+\mu_a[rC'(a)+\frac{(x-Ex\,|a,q_p)f_a(a,q_p)}{\sigma^2 h(q_p)}] \\ &+\mu_{q_p}\left[-\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\right] \\ &+\mu_{q_p}\frac{1}{\sigma^2 h(q_p)}\{x-Ex\,|a,q_p\}\left[f_{q_p}(a,q_p)-1\right] \\ &+\mu_{q_p}\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\frac{1}{\sigma^2 h(q_p)}\left[x-Ex\,|a,q_p\right]^2.\end{aligned} \tag{8}$$

By (8) the compensation scheme is an increasing concave function of the weighted likelihood ratios, and as $h'(q_p)>0$, the right-hand side of (8) is quadratic in x. Since the right-hand side of (8) is a second-degree polynomial for positive μ_{q_p}, the compensation scheme is symmetric around a global minimum and thus low as well as high outcomes are rewarded.[11] Necessarily, the compensation scheme, $s(x)$, is (locally) convex.[12] Now, as

$$s(x) = \frac{1}{r}\ln\left\{\begin{array}{c}\lambda+\mu_a[rC'(a)+\frac{(x-Ex\,|a,q_p)f_a(a,q_p)}{\sigma^2 h(q_p)}] \\ +\mu_{q_p}\left[-\frac{1}{2}h'(q_p)/h(q_p)\right] \\ +\mu_{q_p}\frac{1}{\sigma^2 h(q_p)}\{x-Ex\,|a,q_p\}\left[f_{q_p}(a,q_p)-1\right] \\ +\mu_{q_p}\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\frac{1}{\sigma^2 h(q_p)}\left[x-Ex\,|a,q_p\right]^2\end{array}\right\}+C(a)+\frac{1}{r}\ln(r)$$

it follows that

$$s'(x)=\frac{1}{r}\hat{k}(x)\left(\mu_a\frac{f_a(a,q_p)}{\sigma^2 h(q_p)}+\mu_{q_p}\frac{(f_{q_p}(a,q_p)-1)h(q_p)+(x-f(a,q_p)-(q-q_p))h'(q_p)}{\sigma^2 h(q_p)^2}\right),$$

where

$$\hat{k}(x)=\left[\begin{array}{c}\lambda+\mu_a[rC'(a)+\frac{(x-Ex\,|a,q_p)f_a(a,q_p)}{\sigma^2 h(q_p)}]+\mu_{q_p}\left[-\frac{1}{2}h'(q_p)/h(q_p)\right] \\ +\mu_{q_p}\frac{1}{\sigma^2 h(q_p)}\{x-Ex\,|a,q_p\}\left[f_{q_p}(a,q_p)-1\right] \\ +\mu_{q_p}\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\frac{1}{\sigma^2 h(q_p)}\left[x-Ex\,|a,q_p\right]^2\end{array}\right]^{-1}.$$

[11] Also, the Mirrlees problem no longer exists.

[12] We henceforth assume the compensation contract given by (8) does not violate the limited liability constraint – this turns out to be the case in our examples.

As $\hat{k}(x)$ is the inverse of the agent's marginal utility, it follows that $\hat{k}(x) > 0$, $\forall x$. This implies that the slope of the compensation scheme, when evaluated at the mean $f(a, q_p) + (q - q_p)$, is

$$s'(Ex\,|a, q_p) = \frac{1}{r}\hat{k}(Ex\,|a, q_p)\left(\mu_a \frac{f_a(a, q_p)}{\sigma^2 h(q_p)} + \mu_{q_p}\frac{f_{q_p}(a, q_p) - 1}{\sigma^2 h(q_p)}\right),$$

which is positive since (for any optimal q_p)[13]

$$f_{q_p}(a, q_p) > 1,$$

i.e., the marginal return from investment in the productive technology is larger than the marginal return from investing in the risk-free asset (marginal cost of capital).

As demonstrated by Holmström (1979) it is the informativeness of the outcomes that determines pay. Locally – around the mean – lower outcomes signal the agent slacked off and/or underinvested and thus locally, lower outcomes are considered bad news regarding effort and investment decisions. In contrast, low as well as high outcomes – far from the mean – signal that the agent undertook the (desired) risky investment. That is, the shape of the compensation function is caused by the fact that "extreme" outcomes signal the agent undertook the (desired) risky investment and the location is caused by the fact that around the mean, higher outcomes signal the agent undertook the desired productive effort and investment. As the sign of $s''(x)$ is determined by a second-degree polynomial in x, it follows that the optimal compensation contract takes the form of a "butterfly". That is, $s(x)$ is symmetric, convex in a neighborhood around the minimum, and concave in the tails.

Conveniently it turns out that optimal compensation is fundamentally unaffected by the agent's market alternative.

Proposition 2 *Given interior solutions, the optimal contract is unique up to a positive constant.*

Proof. Let $s(x)$ be an optimal solution for reservation utility level $R = -\exp\{-rR^{CE}\}$. Assume the agent's reservation certainty equivalent is instead $R^{CE} + \Delta$. The agent's certainty equivalent of the compensation scheme $s(x) + \Delta$ is $R^{CE} + \Delta$ and thus the individuel rationality constraint is satisfied. As the agent's preferences display no wealth effects, the incentive compatability constraints are both satisfied. Hence, $s(x) + \Delta$ is feasible. Assume a compensation scheme $\hat{s}(x)$ entails a higher expected net payment to the principal relative to $s(x) + \Delta$. As $\hat{s}(x) - \Delta$ is feasible for reservation certainty equivalent R^{CE}, $s(x)$ cannot be optimal, a contradiction. ■

[13] Also, if the compensation scheme is decreasing around the mean, the agent's first-order condition regarding effort cannot be satisfied.

Note, the result holds regardless of whether the first-order approach is valid. Again, as there are no wealth effects we can rewrite the compensation function:

$$
s(x) = \frac{1}{r}\ln(\lambda) + \frac{1}{r}\ln\left\{\begin{array}{c} 1 + \frac{\mu_a}{\lambda}[rC'(a) + \dfrac{(x - Ex\,|a, q_p\,)f_a(a, q_p)}{\sigma^2 h(q_p)}] \\ +\frac{\mu_{q_p}}{\lambda}\left[-\frac{1}{2}h'(q_p)/h(q_p)\right] \\ +\frac{\mu_{q_p}}{\lambda}\frac{1}{\sigma^2 h(q_p)}\left\{x - Ex\,|a, q_p\right\}\left[f_{q_p}(a, q_p) - 1\right] \\ +\frac{\mu_{q_p}}{\lambda}\frac{1}{2}\frac{h'(q_p)}{h(q_p)}\frac{1}{\sigma^2 h(q_p)}\left[x - Ex\,|a, q_p\right]^2 \end{array}\right\}
$$
$$
+C(a) + \frac{1}{r}\ln(r),
$$

which is convenient for our numerical examples as we can concentrate on $\hat{\mu}_a = \mu_a/\lambda$, and $\hat{\mu}_{q_p} = \mu_{q_p}/\lambda$ and subsequently add a constant to satisfy individual rationality.

4.1 Numerical results

In our numerical examples, we consider a production function of the form

$$
f(a, q_p) = k\left(a^\alpha q_p^\beta\right)^{\frac{1}{\alpha+\beta}},
$$

where k is a general productivity scaling parameter, α is the (not-normalized) marginal productivity of effort, whereas β is the (not-normalized) marginal productivity of investment. We consider the base case parameters in Table 1.

Description	symbol	value
marginal productivity of effort	α	2
marginal productivity of production	β	6
scale of production	k	2.0
agent's absolute risk aversion	r	0.1
variance of random shock	σ^2	10.5
production induced variance of outcome	$h(q_p)$	$q_p^{1/4}$
agent's cost of effort	$C(a)$	a^2

Table 1: Base case parameters for the numerical examples.

Thus, the production function becomes

$$
f(a, q_p) = 2a^{1/4}q_p^{3/4}.
$$

In order to get a picture of the importance of the agency costs, we derive the first best solution in our base case. It turns out that it is optimal to have effort $a = 0.84375$ and investment $q_p = 4.27148$, which yield a value to the principal of 0.711914. This is the principal's value if effort and investment level are contractible. Inasmuch as the agent has no direct cost of productive investment in this case, he does not need to be compensated for undertaking investments. Hence, the principal only has to compensate the agent for undertaking the desired effort level which is obtained by paying the agent a constant salary, such that his individual rationality constraint is satisfied.

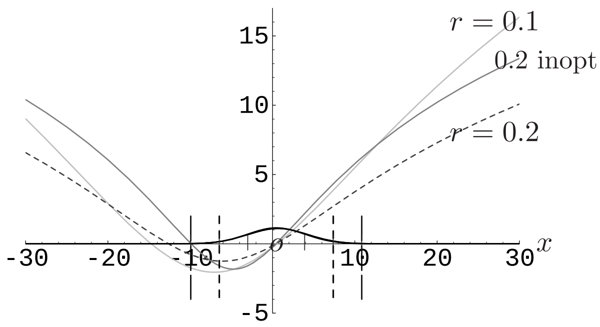
We now consider the second best solution in our base case, i.e. the principal must offer the agent a contract in order to appropriately induce effort and productive investment. The optimal combination of effort and investment choice is $a = 0.400$ and $q_p = 2.35$, respectively, and the second best solution yields a value to the principal equal to 0.378759.[14] Thus, when the principal has to induce effort as well as investments both are significantly decreased and, unsurprisingly, the principal obtains a much lower value. The optimal contract inducing the specified effort and investment decision is depicted as light gray in Figure 2(a) and, as previously argued, the compensation scheme has the butterfly form.

In Figure 2(a) we also depict a case of higher risk aversion, $r = 0.2$. In this case, we plot the optimal contract as well as the "conditional" contract, i.e. we fix $(a, q_p) = (0.4, 2.35)$ from the case of $r = 0.1$. The optimal contract induces $(a, q_p)_{r=0.2} = (0.275, 1.70)$. It is clear that a higher risk aversion makes the agent more sensitive to the uncertainty of the outcome and, hence, the agent must in particular be compensated for undertaking a (high) productive investment level ($q_p = 2.35$) since investments directly influences the variance of the outcome. In the ex ante problem faced by the principal it is therefore better to induce a lower level of effort and investment, which on the one hand decreases the expected outcome, but on the other hand decreases the expected payment to the agent. As a result the increase in the risk aversion decreases the principal's value from 0.378759 to 0.271829.[15] In Figure 2(b) we consider a further increase in the risk aversion and we depict the optimal contracts when $r \in \{0, 1, 0.2, 0.3\}$. In the latter case, it is optimal for the principal to induce an even lower effort and investment level, $(a, q_p) = (0.225, 1.20)$, and his value decreases accordingly to 0.217376.

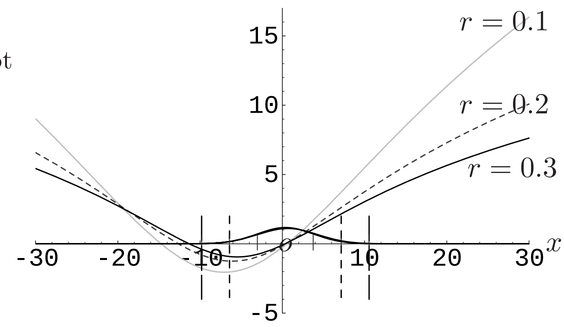
In order to study the effect of the production variance function $h(q_p)$ we consider two choices in Figure 2(c), where we fix effort to $a = 0.25$ and production to $q_p = 1.0$. In the first case, $h(q_p) = \overline{h} = q_p^4$, in the other case $h(q_p) = \underline{h} = q_p^{1/4}$. Thus, given this choice of (a, q_p), the mean (0.414214) and the standard deviation (3.24037) of the distribution of the outcome x is independent of h. In the former case, the principal's value is 0.284945, whereas it is 0.286312 in the latter case. Figure 2(c) reveals that the compensation scheme is very sensitive in the uncertainty induced by risky productive investments, ceteris paribus. Clearly, the more concave

[14] The normalized Lagrange multipliers associated with inducing effort, $a = 0.400$, and investment, $q_p = 2.35$, are $\mu_a/\lambda = 0.34461418$ and $\mu_{q_p}/\lambda = 0.75547044$, respectively.

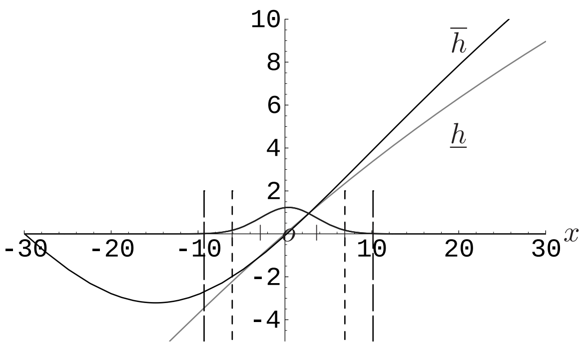
[15] The principal's value with the conditional contract is 0.208232.



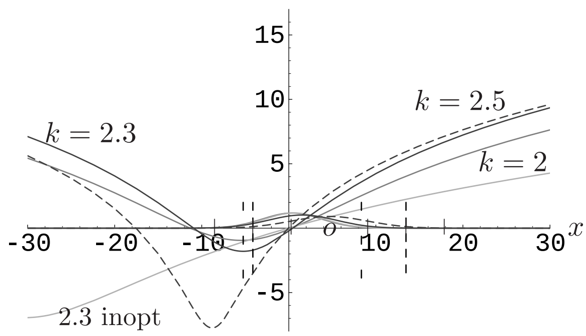
(a) (Conditional) optimal contracts for two degrees of risk aversion, r; the standard deviations are from the case $r = 0.2$ "inopt" marks the case of fixed effort and production



(b) Optimal contracts for various degrees of risk aversion, r; the standard deviations are from the case $r = 0.3$



(c) Contracts for two types of variance functions h. The same pair $(a, q_p) = (0.25, 1.0)$ is induced in both cases.



(d) Contracts for three types of production function scaling k when $r = 0.3$. The vertical lines mark $\pm$ two standard deviation when $k = 2.3$ and $k = 2.5$ (slightly dashed). "inopt" marks the case of fixed effort and production.

Figure 2: Compensation contract, $s(x)$, and the density of the outcome, $g(x|a, q_p)$ (g is scaled by 10). "o" marks the mean of x, "|" marks one unit of standard deviation.

is the production variance function, the less expensive it is for the principal to induce the agent to implement a higher choice of productive investment. The *optimal* contracts for the two choices of h are $(a, q_p)_{h=\overline{h}} = (0.25, 0.80)$ (principal's value $= 0.295934$) and $(a, q_p)_{h=\underline{h}} = (0.40, 2.35)$ (principal's value $= 0.378759$), respectively. Thus, a more concave production variance function makes it optimal for the principal to induce a higher level of productive investments.

In Figure 2(d) we consider the effect of increasing production profitability, i.e. we increase the scaling of the production function, k (we have kept $r = 0.3$). Hence, fixing effort and production, the mean increases while the variance, $h(q_p)\sigma^2$, and the effort cost, $C(a)$, are unchanged. This exercise is specifically plotted as the light gray curve in Figure 2(d). That is, we start out with $k = 2$ where the optimal contract induces $(a, q_p) = (0.225, 1.450)$ and the principal's value is 0.217397 (medium gray curve). In this case the mean is 0.370127 and the standard deviation is 3.39442. We then increase production profitability to $k = 2.3$ but keep effort and production unchanged. Higher production profitability increases the principal's value to 0.528658, the mean to 0.643146 while the standard deviation is unchanged at 3.39442. However, as seen in the figure, the compensation schedule is "shifted" to the left. Now, if we consider the *optimal* contract for $k = 2.3$, then the pair $(a, q_p) = (0.525, 4.50)$ is induced. As a result the mean increases to 1.54891, the standard deviation to 3.91063, the principal's value to 0.907119, and the compensation schedule is "shifted back" (dark curve). Finally, we increase production profitability even further to $k = 2.5$ which yields the optimal compensation schedule plotted as the dashed curve in Figure 2(d). In this case, $(a, q_p) = (1.15, 11.50)$, the mean is 4.66731, the standard deviation is 4.39726, and the principal's value is 2.31283.[16]

After we have seen some of the key comparative static effects, we now turn to consider various restrictions on the compensation.

5 Restrictions on the incentive function

When the investment level is unobservable, low as well as high outcomes are rewarded in order to induce risky investments. In principle, there is nothing wrong with rewarding low outcomes, however, if the agent can destroy output, rewarding low outcomes is not a viable approach, see Dye (1988). Also, rewarding high as well as low outcomes may invite overinvestment. To see this, consider that compensation looks as in Figure 3.

[16] Of course, the first best solution is independent of variations in risk aversion and variance. However, k directly influences the production profitability and, hence, changes the first best solution. When $k = 2.3$ we obtain $(a, q_p)_{FB} = (1.47572, 13.0666)$ and the principal's value is 2.17776 and when $k = 2.5$ we obtain $(a, q_p)_{FB} = (2.05994, 25.46)$ and the principal's value is 4.24334.

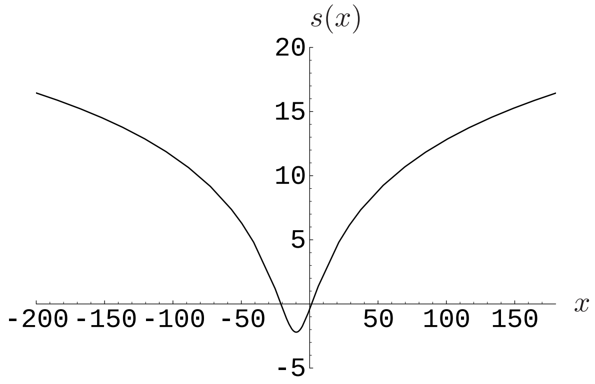


Figure 3: Compensation contract, $s(x)$.

Hence, it is obvious that the agent prefers to overinvest if overinvestment leads to extreme variance. Also, overinvestment is preferred if it leads to low expected outcome, i.e., when overinvestment – in expectation – constitutes destruction of outcome.[17] Thus, if the agent can access the capital market without the principal observing it, the contract cannot be symmetric as depicted – though it could well be decreasing on some interval(s). If we solve the problem assuming non-decreasing compensation, the compensation scheme will look as in Figure 4 (the multipliers are not necessarily identical to the multipliers in the solution where the slope is unrestricted).

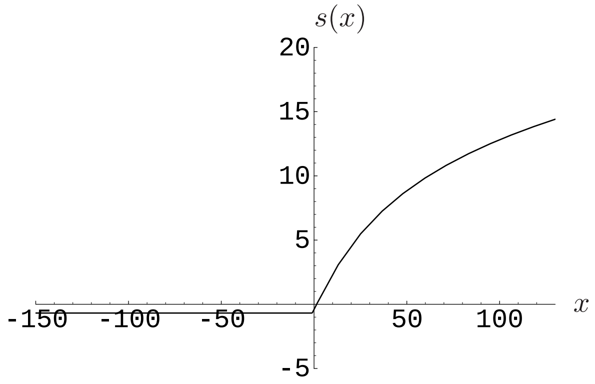


Figure 4: Compensation contract, $s(x)$, in the case of non-negative slope.

Depending on where the endpoint of the flat part of the compensation function is located, the strictly increasing part can be both locally convex and locally concave. In this case it is not the limited liability constraint which introduces flat spots in the contract. Instead it is caused either by the fact the agent can destroy output or by the fact the agent can overinvest.

[17] That is, although he agent's problem is locally concave around the optimal effort and investment levels identified in (1), the agent's problem is generally unbounded in q_p given an incentive scheme of the type depicted in figure 1.

Regardless, the contract is still (locally) convex.

6 Piecewise linear compensation functions

Option contracts resemble the optimal non-decreasing contracts. However, contrary to an option contract the optimal nondecreasing contract is concave in the upper tail. Still, it is potentially interesting to assess the loss in efficiency connected to issuing options instead of offering an optimal contract. If we assume a piecewise linear compensation function of the form

$$s(x) = \begin{cases} \underline{s}' & ; x \leq \dfrac{\underline{s}' - \kappa_0}{\kappa_1}, \\ \kappa_0 + \kappa_1 x & ; x > \dfrac{\underline{s}' - \kappa_0}{\kappa_1}, \end{cases}$$

then the agent's expected utility is

$$\begin{aligned} E[U(s(x);a);q_p] &= \int_{-\infty}^{\frac{\underline{s}'-\kappa_0}{\kappa_1}} -\exp\{-r(\underline{s}' - C(a))\} g(x\,|a,q_p)\,dx \\ &+ \int_{\frac{\underline{s}'-\kappa_0}{\kappa_1}}^{\infty} -\exp\{-r(\kappa_0 + \kappa_1 x - C(a))\} g(x\,|a,q_p)\,dx. \end{aligned}$$

Now,

$$\begin{aligned} & \exp\{-r(\kappa_0 + \kappa_1 x - C(a))\} g(x\,|a,q_p) \\ = \; & \exp\{-r(\kappa_0 + \kappa_1 x - C(a))\} \frac{1}{h(q_p)^{\frac{1}{2}}\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2}\frac{(x - f(a,q_p) - (q-q_p))^2}{\sigma^2 h(q_p)}\} \\ = \; & \exp\{-r(\kappa_0 + \kappa_1[f(a,q_p) + (q-q_p)] - \frac{1}{2} r\kappa_1^2\sigma^2 h(q_p) - C(a))\} \\ & \times \frac{1}{h(q_p)^{\frac{1}{2}}\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2}\frac{(x - [f(a,q_p) + (q-q_p) - r\kappa_1\sigma^2 h(q_p)])^2}{\sigma^2 h(q_p)}\}, \end{aligned}$$

which implies that

$$\begin{aligned} & E[U(s(x);a);q_p] = \\ & -\exp\{-r(s' - C(a))\} \int_{-\infty}^{\frac{\underline{s}'-\kappa_0}{\kappa_1}} \frac{1}{h(q_p)^{\frac{1}{2}}\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2}\frac{(x - f(a,q_p) - (q-q_p))^2}{\sigma^2 h(q_p)}\} dx \\ & -\exp\{-r(\kappa_0 + \kappa_1[f(a,q_p) + (q-q_p)] - \frac{1}{2} r\kappa_1^2\sigma^2 h(q_p) - C(a))\} \\ & \times \int_{\frac{\underline{s}'-\kappa_0}{\kappa_1}}^{\infty} \frac{1}{h(q_p)^{\frac{1}{2}}\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2}\frac{(x - [f(a,q_p) + (q-q_p) - r\kappa_1\sigma^2 h(q_p)])^2}{\sigma^2 h(q_p)}\} dx. \end{aligned}$$

That is, expected utility can be written as

$$\begin{aligned} E[U(s(x);a);q_p] &= U(\underline{s};a) P\big(x \leq \frac{\underline{s}' - \kappa_0}{\kappa_1}\big) \\ &+ U(\kappa_0 + \kappa_1[f(a,q_p) + (q-q_p)] - \frac{1}{2} r\kappa_1^2\sigma^2 h(q_p); a) P\big(y \geq \frac{\underline{s}' - \kappa_0}{\kappa_1}\big), \end{aligned}$$

where

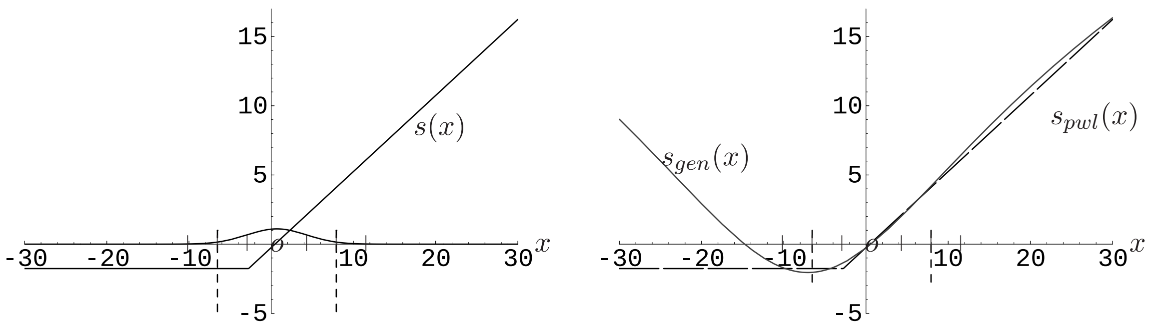
$$y \sim N(f(a,q_p) + (q-q_p) - r\kappa_1\sigma^2 h(q_p); h(q_p)\sigma^2).$$

6.1 Numerical results

Let us again turn to our base case from Table 1. We now employ the restricted contract – i.e. an option-like contract – in our base case example. Solving the second best case yields that the following restricted contract is "optimal":

$$s(x) = \begin{cases} -1.76421 & ; x \leq -2.75485 \\ -0.25036 + 0.549522x & ; x > -2.75485 \end{cases}.$$

This contract is depicted in Figure 5(a). The contract induces an effort level of $a = 0.410$ and an investment level of $q_p = 2.40$. The induced pair of effort and productive investment is close to the solution with the unrestricted contract. Comparing the optimal unrestricted contract to the optimal piecewise linear contract, which is done Figure 5(b), shows that the compensation schemes are quite similar in the two cases when we consider the outcome region defined by ± 2 times the standard deviation around the mean. Employing the optimal unrestricted contract yields an expected compensation cost of 0.290121, whereas the optimal piecewise linear contract carries an expected cost of 0.307849. In the option-like case the principal's value is 0.378069 which is only slightly lower than the principal's value, 0.378759, from the unrestricted case. Hence, at lest in this particular example it seems not much is lost by restricting the contractual form.



(a) Compensation contract $s(x)$ and the density of the outcome g (scaled by 10). "o" marks the mean of x, "|" marks one unit of standard deviation.

(b) Compensation contracts, $s_{gen}(x)$ for the unrestricted contract and $s_{pwl}(x)$ (dashed) for the case of a piecewise linear contract.

Figure 5: Compensation schemes in the case of a piecewise linear (option-like) contract. The base case parameters from Table 1 are used.

6.2 Incentive measures

The recent financial literature on the valuation of executive stock options often consider how to measure incentives. A commonly used measure is the sensitivity of the option value or the certainty equivalent to changes in current stock price.

In order to study the validity of the above measure, we consider the following exercise. Previously we have solved for the optimal piecewise linear contract which induces the manager to undertake $(a, q_p) = (0.41, 2.4)$, see Figure 5. We now allow the principal to offer the agent another option-like contract with a higher slope. This alternative contract should provide the manager with better incentives according to the commonly used incentive measure. The contracts are depicted in Figure 6, where the former contract is denoted "optimal" and an example of the latter contract is denoted "inoptimal". Clearly, the latter contract has higher incentives as measured by the slope. Also, the latter contract is designed such that (i) both contracts become increasing at the same level of output and (ii) *if* the agent undertakes $(a, q_p) = (0.41, 2.4)$, then his expected utility is equal to his reservation utility $(= -1)$. If the incentive measure is valid, the agent should undertake an effort and productive investment choice such that, at the very least, the principal is not worse off with the alternative contract. Now, if the principal provides the agent with the alternative contract, the agent undertakes $(a, q_p) = (0.7281, 3.74)$. Thus, the agent increases effort and productive investments. In this case, the agent has an expected utility of -0.989587 which is higher than his reservation utility equal to -1. Interestingly, the expected output is $E(x|a = 0.7281, q_p = 3.74) = 1.23$, whereas the contract solved for in the previous section yielded a *lower* expected output of $E(x|a = 0.41, q_p = 2.4) = 0.69$. However, the expected compensation to the agent is *higher* in the case of the contract with the high slope (0.308 versus 1.26). The increase in the expected compensation to the agent more than trade offs the increase in the expected outcome. In other words, albeit the slope is higher (and the expected outcome is higher, too), i.e. the incentives are measured to be higher with the commonly used measure, the alternative contract is inoptimal for the principal.[18]

7 Additional information

In the analytical literature concerning models of executive compensation it is widely acknowledged that remuneration should contain a relative performance element. Holmström (1982) predicts that systematic risk will be filtered out through relative performance evaluation and a similar conclusion is reached by Mookherjee (1984), Dye (1992), and Demski and Sappington

[18] In fact, the principal's value decreases from 0.378 to -0.034.

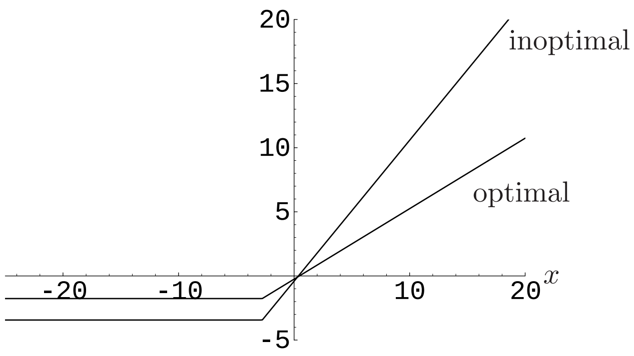


Figure 6: Piecewise linear compensation contracts: "optimal" is the optimal contract inducing $(a, q_p) = (0.41, 2.4)$. The contract "inoptimal" has a higher slope, but the contract is not optimal.

(1984). Given models such as the Capital Asset Pricing Model and the Arbitrage Pricing Theory one would expect equity incentives to display an element of relative performance evaluation. For example Abowd and Kaplan (1999) make the observation:

"Stock options reward stock price appreciation regardless of the performance of the economy or sector. ... If the exercise price could be linked to measures like the S&P 500, or an index of close product-market competitors, then executives could be rewarded for gains in stock price in excess of those explainable by market factors outside their control. If market-wide stock movements could be netted out of executive incentive schemes, then equivalent incentives could be provided while reducing the volatility of executive's portfolios." (p.162)

The underlying argument is that relative performance evaluation reduces the risk premium and thus is efficient. While relative performance evaluation seems relatively uncommon it might well be taking place. One possibility is the agent via his personal account hedges the systematic risk component in his compensation. Another possibility is the relative performance evaluation takes place through the unobservable part of the incentive arrangement – layoff decisions, future option grants, and discretionary fringe benefits could depend on relative performance.

Holmström (1979) determines the necessary and sufficient condition for relative performance evaluation to be efficiency enhancing, and although the condition contains little predictive power as to *how* additional information should enter, it is very precise regarding *what* information should enter the performance contract. As hitherto we assume

$$\tilde{x} = f(a, q_p) + h(q_p)^{\frac{1}{2}}\tilde{\epsilon} + q - q_p.$$

However, in addition we assume $\tilde{\epsilon} = \tilde{\theta} + \tilde{\gamma}$ where $\tilde{\theta}$ is a random component affecting the market or the particular industry in which the company is operating and $\tilde{\gamma}$ is idiosyncratic noise.

Furthermore, $\tilde{\theta} \sim N(0, \sigma_\theta^2)$, $\tilde{\gamma} \sim N(0, \sigma_\gamma^2)$, and $cov(\tilde{\theta}, \tilde{\gamma}) = 0$. Similarly, let

$$\tilde{y} = \tilde{\theta} + \tilde{\xi}$$

be the unanticipated change in the value of the market portfolio, the unanticipated change in a competitor's market value, or the unanticipated change in an industry index, where $\tilde{\xi} \sim N(0, \sigma_\xi^2)$, where $cov(\tilde{\theta}, \tilde{\xi}) = 0$, and where $cov(\tilde{\gamma}, \tilde{\xi}) = 0$.[19] It follows that

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \sim N\left(\begin{matrix} E\tilde{x}\,|a, q_p \\ E\tilde{y} \end{matrix} , \Sigma(q_p) \right),$$

where

$$\Sigma(q_p) = \begin{bmatrix} h(q_p)(\sigma_\theta^2 + \sigma_\gamma^2) & h(q_p)^{\frac{1}{2}}\sigma_\theta^2 \\ h(q_p)^{\frac{1}{2}}\sigma_\theta^2 & \sigma_\theta^2 + \sigma_\xi^2 \end{bmatrix}.$$

The joint density function is:

$$\begin{aligned} g(x, y\,|a, q_p) &= (2\pi)^{-\frac{1}{2}}\,|H(q_p)|^{\frac{1}{2}} \\ &\quad \exp\{-\frac{1}{2}\begin{pmatrix} x - E\tilde{x}\,|a, q_p \\ y - E\tilde{y} \end{pmatrix}^t H(q_p) \begin{pmatrix} x - E\tilde{x}\,|a, q_p \\ y - E\tilde{y} \end{pmatrix}\}, \end{aligned}$$

where $H(q_p) = \Sigma(q_p)^{-1}$ is the 2×2 precision matrix and $|H(q_p)|$ denotes the determinant. It follows that

$$\begin{aligned} \frac{g_a(x, y\,|a, q_p)}{g(x, y\,|a, q_p)} &= \frac{\partial}{\partial a}\left\{ -\frac{1}{2}\begin{pmatrix} x - E\tilde{x}\,|a, q_p \\ y - E\tilde{y} \end{pmatrix}^t H(q_p) \begin{pmatrix} x - E\tilde{x}\,|a, q_p \\ y - E\tilde{y} \end{pmatrix} \right\} \\ &= \frac{1}{|\Sigma(q_p)|} f_a(a, q_p)\left[(x - f(a, q_p) - (q - q_p))(\sigma_\theta^2 + \sigma_\xi^2) - h(q_p)^{\frac{1}{2}}\sigma_\theta^2 y \right] \\ &= \frac{1}{|\Sigma(q_p)|} f_a(a, q_p)(\sigma_\theta^2 + \sigma_\xi^2)\left[x - E\tilde{x}\,|a, q_p, y \right], \end{aligned}$$

and that

$$\begin{aligned} \frac{g_{q_p}(x\,|a, q_p)}{g(x\,|a, q_p)} &= -\frac{1}{2}h'(q_p)/h(q_p) \\ &\quad + \frac{1}{|\Sigma(q_p)|}(f_{q_p}(a, q_p) - 1)\left\{ (x - E\tilde{x}\,|a, q_p)(\sigma_\theta^2 + \sigma_\xi^2) - h(q_p)^{\frac{1}{2}}\sigma_\theta^2 y \right\} \\ &\quad + \frac{1}{2}\frac{1}{|\Sigma(q_p)|}\frac{h'(q_p)}{h(q_p)}(x - E\tilde{x}\,|a, q_p)\left\{ (x - E\tilde{x}\,|a, q_p)(\sigma_\theta^2 + \sigma_\xi^2) - h(q_p)^{\frac{1}{2}}\sigma_\theta^2 y \right\} \\ &= -\frac{1}{2}h'(q_p)/h(q_p) \\ &\quad + \frac{1}{|\Sigma(q_p)|}(\sigma_\theta^2 + \sigma_\xi^2)(f_{q_p}(a, q_p) - 1)(x - E\tilde{x}\,|a, q_p, y) \\ &\quad + \frac{1}{2}\frac{1}{|\Sigma(q_p)|}\frac{h'(q_p)}{h(q_p)}(\sigma_\theta^2 + \sigma_\xi^2)(x - E\tilde{x}\,|a, q_p)(x - E\tilde{x}\,|a, q_p, y). \end{aligned}$$

[19] If the interpretation is that of a competitor, the competitor's payoff, $\tilde{x}_C$, is given by $\tilde{x}_C = \mu_C + \tilde{\theta} + \tilde{\xi}$.

It is obvious that the additional information, y, enters the compensation function and from (8) it is known that compensation is monotone in the weighted sum $\mu_a g_a/g + \mu_{q_p} g_{q_p}/g$. Locally around the conditional mean, $E\tilde{x}\,|a, q_p, y$, higher outcomes are still considered a signal the agent worked hard and invested as instructed. Hence, the agent is clearly benchmarked against peer or market performance, y. Immediately one might expect the squared difference between observed outcome and conditional mean would enter compensation as in (8). This turns out to be erroneous. Compensation depends on the difference between outcome and the unconditional mean multiplied by the difference between outcome and the conditional mean,

$$(x - E\tilde{x}\,|a, q_p)(x - E\tilde{x}\,|a, q_p, y).$$

Outcomes between the two means are considered bad news, whereas outcomes that are extreme relative to the two means are considered good news. Assume for the moment $E\tilde{x}\,|a, q_p, y > E\tilde{x}\,|a, q_p$. The interpretation of the benchmark, $y > 0$, is the common noise component, $\tilde{\theta}$, is positive

$$E\tilde{\theta}\,|y = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\xi^2} y.$$

Given the common noise component is positive, the expected deviation from the unconditional mean is

$$Eh(q_p)^{\frac{1}{2}}\tilde{\theta}\,|y = \frac{h(q_p)^{\frac{1}{2}}\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\xi^2} y.$$

Hence, an outcome above the unconditional but below the conditional mean is a sign investment $(h(q_p)^{\frac{1}{2}})$ is too low. Extreme values above the conditional mean are still considered a sign the agent invested, as are outcomes below the unconditional mean. Benchmarking is clearly taking place, however, given any y truly extreme firm specific outcomes are still considered good news concerning investment. One message in Holmström (1979) is the additional information, y, should enter the performance evaluation, provided x is not a sufficient statistic for x and y with respect to effort and investment. In our situation this seems a relatively innocent result, however, it has far reaching implications. The agent should be benchmarked against his peers and/or market performance and compensation should contain an element that depends on the product between the conditional and the unconditional mean. It might be possible for the agent to take care of the benchmarking part on his personal account, however, it is harder to see how the agent could deal with the latter part of the performance evaluation statistic - or even be given incentives to do so. Thus it seems Abowd and Kaplan (1999) have a point.

8 Conclusion

We study how the firm's equity holders should optimally reward the the manager for his work. A central question in our paper is to consider when it is optimal for the firm owners to provide the manager with an option-like contract. Empirically, such contracts play an important role, but a number of recent papers dispute whether the granting of option compensation to managers is worth the equity holders' while.

In order to analyze optimal compensation we apply a principal-agent setting. However, in the standard principal-agent framework, it is only the manager's choice of action or effort which is non-contractible. In our version of the standard setting, it is a limited liability constraint which introduces a convexity-like feature in the manager's contract. We argue, however, that a key element in optimal compensation is to take the manager's investment problem into account. Therefore, we extend the principal agent model to include effort as well as investment choice by the manager. The introduction of a productive investment choice leads to an induced moral hazard problem, because the equity holders have a dichotomy in providing the manager with the appropriate incentives for choosing effort and investment.

In our setting we show that it is not necessary to impose limited liability on the manager in order to obtain optimal convex-like compensation. As a function of the firm's output, the optimal compensation to the manager is increasing in an interval surrounding the expected output. However, since a sufficiently bad outcome is a sign that the manager undertook a high desired production choice, the manager's compensation is decreasing for low outcomes. If the manager can destroy output (or overinvest), it is necessary to let compensation be non-decreasing in output. In that case, the shape of the restricted (optimal) compensation function is closely related to providing the manager with a call-option on output. We further restrict the compensation to be piecewise linear. In our numerical examples, we illustrate that this additional restriction does not seem to be very costly ex ante. That is, we have provided a rationale for option compensation when the manager also undertakes a productive investment choice.

Furthermore, in the recent financial literature on equity based pay, an often applied measure of the manager's incentives is the sensitivity between the manager's certainty equivalent and the current output (the stock price). In an example, we illustrate that incentive measures along such lines may provide the wrong insight as regards the manager's incentives. In fact, providing stronger "incentives" with the often used financial incentive measure turns out to lead to inoptimal effort and investment choice. This is costly ex ante.

Finally, we also consider the effect of additional information such that output noise stems from market risk and idiosyncratic risk. This has implications for benchmarking the agent. We demonstrate that the agent should be benchmarked against market performance (or his peers) and, in addition, the compensation to the agent should contain an element depending on the product between the conditional and the unconditional mean. That is, optimal benchmarking is potentially quite complicated.

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