Explaining Volatility Smiles of Equity Options with Capital Structure Models

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Abstract

We investigate the behavior of prices of equity options in the Corporate Securities Framework suggested by Ammann and Genser (2004) where equity is the residual claim of a stochastic EBIT. Option prices are obtained by two numerical methods: (i) Approximation of the EBIT-process by a trinomial lattice and calculation of all securities by backward induction. (ii) Evaluation of the risk-neutrally expected value of the equity option at maturity by direct numerical integration. Economically, the current state of the firm with respect to bankruptcy and the capital structure influence to a large extent the particular risk-neutral equity (return) distribution at option maturity and the level and slope of implicit Black and Scholes (1973)-volatilities as a function of strike prices. Additionally, we oppose the tradition of relating equity return moments to implied volatilities. This connection might be misleading

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We investigate the behavior of prices of equity options in the Corporate Securities Framework suggested by Ammann and Genser (2004) where equity is the residual claim of a stochastic EBIT. Option prices are obtained by two numerical methods: (i) Approximation of the EBIT-process by a trinomial lattice and calculation of all securities by backward induction. (ii) Evaluation of the risk-neutrally expected value of the equity option at maturity by direct numerical integration. Economically, the current state of the firm with respect to bankruptcy and the capital structure influence to a large extent the particular risk-neutral equity (return) distribution at option maturity and the level and slope of implicit Black and Scholes (1973)-volatilities as a function of strike prices. Additionally, we oppose the tradition of relating equity return moments to implied volatilities. This connection might be misleading when bankruptcy probabilities become high.

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1 Introduction

One of the frequently discussed issues in the asset pricing literature is why the theoretical option prices in the Black and Scholes (1973)/Merton (1974) framework cannot be observed empirically. Implied volatilities of observed option prices calculated in the Black/Scholes setting are not constant. They are higher for lower strike prices than for higher ones. The convex relationship is commonly referred to as an option's volatility smile or smirk.[1]

[1] The literature on implied option volatilities is huge. Early evidence of the existence of implied volatility smiles is MacBeth and Merville (1979) who base their work on studies of Latané and Rendleman (1975) and Schmalensee and Trippi (1978). Emanuel and MacBeth (1982) try to relax the stringent volatility assumption of the Black and Scholes (1973) to account for the volatility smile but fail by a constant elasticity of variance model of the stock price. More recent studies of Rubinstein (1994), Jackwerth and Rubinstein (1996), and Jackwerth and Rubinstein (2001) take the volatility smile as given and exploit option prices to extract implied densities of the underlying asset. See also Buraschi and Jackwerth (2001) who report that after the 1987 market crash, the spanning properties of options decreased. They conclude that more assets are needed for hedging option prices which hints to additional risk factors such as stochastic volatility.

Several extensions of the Black/Scholes framework have been suggested to account for these empirical observations which can be categorized in two groups. First, a pragmatic stream of the literature introduced volatility structures and thus changed the physical distributional assumptions for the underlying. Although this procedure yields satisfying results for equity option trading, economic intuition is still lacking which underpins the use of volatility structures.[2]

Second, an economic stream of the literature tried to explain why the pricing kernel, defined as the state price function at option maturity, is different to the one implied by the Black and Scholes (1973) model. Arrow (1964) and Debreu (1959), Rubinstein (1976), Breeden and Litzenberger (1978), Brennan (1979) and others relate state prices to investor utility and the state dependent payoffs of securities and disentangle the effect of the investor's utility function from the probability distribution of the underlying asset. Therefore, the pricing kernel – and the value of securities – depends on assumptions about the utility function of the investor and on the distribution of the security.[3] The pricing kernel is valid for all securities in an economy. However, when taking an individual firm's perspective, the simple pricing kernel needs to be augmented by additional risk factors such as default or liquidity.

Using the Corporate Securities Framework of Genser (2005a) and the analytical solution of Ammann and Genser (2005) and Genser (2005b) a simpler economic explanation might be suggested: The implied Black/Scholes-volatility smile of equity options can be related to the specific ability of equity holders to declare bankruptcy. This feature introduces dependence on the particular path of EBIT and changes the distribution of equity due to the conditioning on survival until option maturity. Contract design of equity, especially the limited liability, changes the local volatility of equity which in turn depends on the current state of the firm and influences the pricing kernel and equity option's implied volatilities. Our approach is related to Geske (1979)'s compound option approach. However, in contrast to Geske (1979) whose underlying of the compound option might be interpreted as a Merton (1974)-like firm with only one finite maturity zero bond outstanding, the Corporate Securities Framework allows for

[2]See e.g. Rubinstein (1994) who adopts this procedure to the binomial model by allowing an arbitrary distribution of the underlying at option maturity. Heston and Nandi (2000) derive closed form solutions for options where the volatility of the underlying which follows a GARCH process.

[3]Franke, Stapleton and Subrahmanyam (1999) analyze the pricing kernel directly by comparing pricing kernels of investors with changing degree of risk aversion at different levels of investor's wealth. Franke et al. (1999) find the pricing kernel to be convex to accommodate for the dependence of risk aversion and investor's wealth leading to convex implied Black/Scholes-volatilities.

a complex capital structure. As will be shown later, the debt structure influences the slope and the level of the implied volatility smile.[4]

In a related empirical study, Bakshi, Kapadia and Madan (2003) link the volatility smile to the distribution of equity returns. They show that a higher skewness and a lower kurtosis of equity returns result in steeper volatility smiles. Empirical evidence supports their hypothesis. However, Bakshi et al. (2003) do not offer an economic explanation why individual stock's returns should be skewed. In our EBIT-based firm value framework, the leverage ratio depends on the current state of the firm with respect to bankruptcy. Firms far from bankruptcy and with low leverage ratios exhibit a risk-neutral equity distribution that reflects the properties of the assumed EBIT-process. The function of implied equity option volatilities with respect to strike prices are at a low level but steep. The closer the firm is to bankruptcy skewness and kurtosis of equity values increase. The implied volatility level rises significantly but the smile becomes flatter, at least in the ABM-case. However, we stress that higher moments of equity returns might be misinterpreted in the presence of bankruptcy probabilities. Moreover, a key determinant of implied volatility structures is the firm's capital structure.

Toft and Prucyk (1997) value equity options in the restrictive Leland (1994) framework. The Corporate Securities Framework extends the Toft and Prucyk (1997) analysis. Their model is a special case of our framework if we restrict the capital structure to perpetual debt, the tax structure to corporate taxes only, and if we assume that free cash flow after taxes follows a geometric Brownian motion instead of EBIT following an arithmetic or geometric Brownian motion. We are able to analyze the pricing of options under different assumptions for the EBIT-process and of firms that have a complex capital structure. Toft and Prucyk (1997) shed some light an complex capital structures when they proxy a debt covenant in the perpetual debt case by the amount of a firm's short term debt. Our model allows to analyze firms with short term debt and long term debt directly.

Although security prices have analytical solutions as shown in Genser (2005b), options written on the equity value can only be valued numerically. We propose two numerical methods which will be used in this section: first, a trinomial lattice is used to approximate the EBIT-process and equity options are valued by backward induction. Second, we calcu-

[4] One might argue that these observations do not carry over to index options because the index cannot go bankrupt. However, the index consists of firms that can go bankrupt. The bankruptcy probability cannot be diversified away. E.g. pick an index of two firms with uncorrelated bankruptcy probabilities of 1 % per year. Only with a probability of 98.01 % both firm survive the next year. With almost 2 % probability the index exhibits at least one bankruptcy.

late the expected option value at option maturity by numerical integration which is possible because we investigate European style equity options. Additionally, both methods allow a fairly good approximation of the first four central moments of the equity value and its return distribution at option maturity. Therefore, we can directly compare our simulation results to empirical findings of Bakshi et al. (2003).

The paper is structured as follows. Section 2 summarizes the the EBIT-framework and describes its approximation in a trinomial lattice. Section 3 explains the valuation of options in the numerical integration scheme. In Section 4 we analyze the simulation results with respect to the equity return distribution and volatility smiles. Section 5 concludes.

2 A Trinomial Lattice Approach for the Corporate Securities Framework

2.1 The Approximation of the EBIT-Process

The stochastic factor in the corporate security framework is the firm's EBIT η which is assumed to follow an arithmetic Brownian motion under the equivalent risk-neutral martingale measure $\mathbb{Q}$

$$d\eta \quad = \quad (\mu_\eta - \theta \cdot \sigma_\eta)\, dt + \sigma_\eta dz^{\mathbb{Q}}, \tag{1}$$

where μ_η and σ_η are the physical drift and standard deviation of the EBIT-process, θ is a risk premium which transforms the original process into a risk-neutral $\mathbb{Q}$-martingale. The EBIT-process is driven by a a standard Wiener process $z^{\mathbb{Q}}$ under the risk-neutral probability measure $\mathbb{Q}$.[5] All parameters in equation (1) are assumed to be constant.

A risk-neutral valuation tree can be constructed in which the discount rate is the riskless interest rate r. We approximate the stochastic process numerically by a trinomial tree with time steps Δt and EBIT step size

$$\Delta\eta \quad = \quad \lambda\sqrt{\sigma_\eta \Delta t}. \tag{2}$$

λ denotes a EBIT spacing parameter.

Writing for the risk-neutral drift of the EBIT process $\mu = \mu_\eta - \theta \cdot \sigma_\eta$, the probabilities at each node to reach the following up-, middle-, and

[5] We assume that all probability spaces are well defined and that all expectations and integrals exist.

down-state nodes are

$$\pi_u = \frac{\sigma_\eta^2 \Delta t + \mu^2 \Delta t^2}{2\Delta\eta^2} - \frac{\mu\Delta t}{2\Delta\eta}, \tag{3a}$$

$$\pi_m = 1 - \frac{\sigma_\eta^2 \Delta t + \mu^2 \Delta t^2}{\Delta\eta^2}, \tag{3b}$$

$$\pi_d = \frac{\sigma_\eta^2 \Delta t + \mu^2 \Delta t^2}{2\Delta\eta^2} + \frac{\mu\Delta t}{2\Delta\eta}. \tag{3c}$$

In equations (3) the two parameters λ and Δt can be chosen freely.

To have a good approximation of the EBIT-process 900 to 1'100 steps are needed that determine Δt subject to the maturity of the tree. Kamrad and Ritchken (1991, p. 1643) suggest a value of $\lambda = 1.2247$ which they show to have the best convergence properties on average in their application in multi-state variables option pricing.

In the Corporate Securities Framework, the firm declares bankruptcy whenever total firm value V hits a constant barrier V_B. The total firm value can be calculated explicitly by discounting all future EBIT with the constant riskless interest rate r which yields

$$V = \frac{\mu}{r^2} + \frac{\eta_t}{r}. \tag{4}$$

EBIT is distributed to all claimants of the firm. So, total firm value of an EBIT-model does not only include the market value of debt and equity, but also bankruptcy losses, and taxes to the government. This alters the notion of the bankruptcy barrier, as well as losses in case of bankruptcy compared to traditional firm value models. The bankruptcy barrier can equivalently be defined in terms of an EBIT-value by

$$\eta_B = V_B \cdot r - \frac{\mu}{r}. \tag{5}$$

The literature on barrier options valuation in lattice models, observes pricing problems because the barrier usually lies between two nodes. If Δt is decreased, the value of the barrier option might not converge because the barrier usually changes its distance to adjacent nodes. Boyle and Lau (1994) demonstrate the oscillating pattern of convergence. To overcome the deficiency Boyle and Lau (1994) suggest to adjust the steps $\Delta\eta$ such that the barrier is positioned just above one layer of nodes.[6] To mimic the algorithm, a λ closest to 1.225 is chosen to ensures that the $]t_0, T_1]$-bankruptcy barrier lies just above one node level. With this parameter

[6]See e.g. Kat and Verdonk (1995) or Rogers and Stapleton (1998) for other methods to overcome the convergence problem.

constellation, the security values of the debt and equity issues converge sufficiently well to their analytical solutions.

The case of geometric Brownian motion can be treated along the same lines. Assume that EBIT follows[7]

$$\frac{d\bar{\eta}}{\bar{\eta}} = \left(\bar{\mu}_\eta - \bar{\theta} \cdot \bar{\sigma}_\eta\right) dt + \bar{\sigma}_\eta dz^{\mathcal{Q}}. \tag{6}$$

where $\bar{\mu}_\eta$ and $\bar{\sigma}_\eta$ denote the constant instantaneous drift and volatility of the process, under the physical measure $\mathcal{P}$, and $\bar{\theta}$ the risk premium to change the measure to the equivalent martingale measure $\mathcal{Q}$. To simplify notation, denote the risk-neutral drift by $\bar{\mu} = \bar{\mu}_\eta - \bar{\theta} \cdot \bar{\sigma}_\eta$. Then, total firm value amounts to[8]

$$\bar{V}_t = \frac{\bar{\eta}_t}{r - \bar{\mu}}, \tag{7}$$

which replaces the version for the arithmetic Brownian motion of equation (4). Then, the logarithm of the EBIT $\bar{\eta}$

$$d\ln(\bar{\eta}) = \left(\bar{\mu} - \frac{\bar{\sigma}_\eta^2}{2}\right) dt + \bar{\sigma}_\eta dz^{\mathcal{Q}}, \tag{8}$$

follows the arithmetic Brownian motion of equation.

2.2 Payments to Claimants and Terminal Security Values

In each node, EBIT is distributed among the claimants of the firm (see Figure 1). Payments to claimants are different in case of bankruptcy. Therefore, Figure 1 exhibits two separate EBIT-distribution algorithms for the firm being (i) solvent and (ii) insolvent. The bankruptcy decision is modeled by testing if the expected future firm value in the current node is lower or equal to the bankruptcy level V_B.[9]

If the firm is solvent, the claims to EBIT are divided between debt and equity investors. Debt holders receive the contracted coupon payments and at maturity the notional amount. The rest of the EBIT remains with the firm.

[7] Trying to keep the notation comparable to the case of arithmetic Brownian motion, we will use a bar to indicate respective GBM-parameters.

[8] See e.g. Shimko (1992) for guidance how to solve for the firm value.

[9] This decision criterion can be generalized to equity holder's optimal decisions whether to honor current and future obligations of other claimants of the firm. The general splitting procedure of EBIT among claimants is not affected.

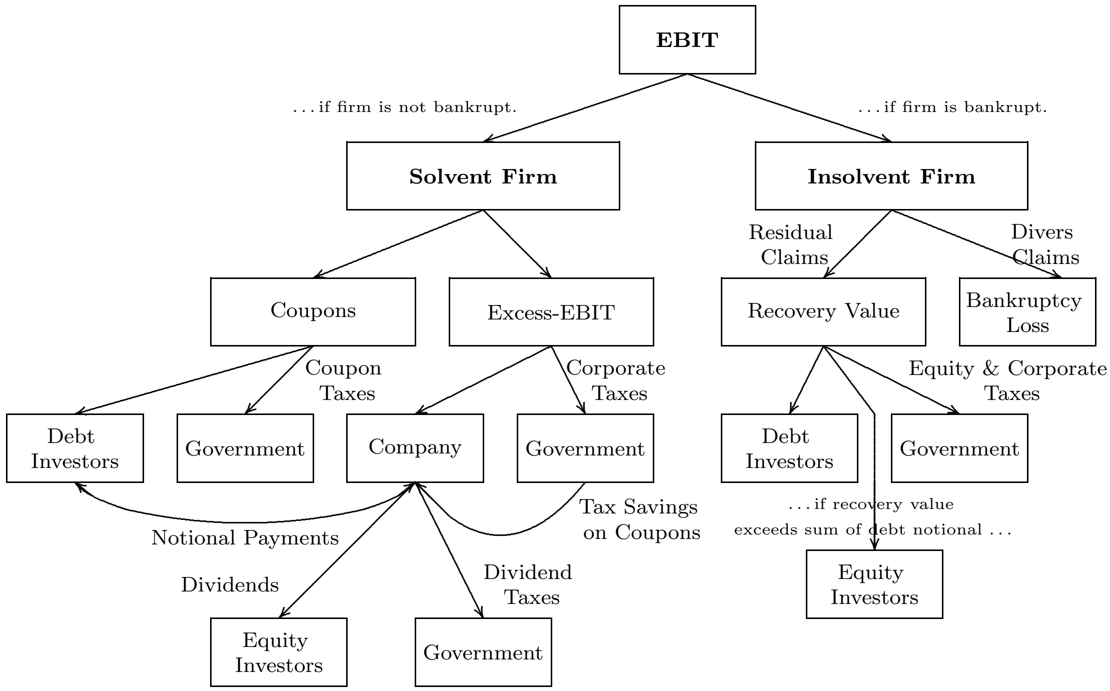

Figure 1: Splitting the EBIT in a typical node among claimants

The government imposes a tax regime which reduces the payments to the different security holders further. Debt investors pay a personal income tax rate τ^d on their coupon income. The firm pays a corporate income tax rate τ^c on corporate earnings – EBIT less coupon payments. What remains after adjusting for cash flows from financing transactions – the issue of new debt or the repayment of old debt – is paid out to equity investors as a dividend. The dividend is taxed at a personal dividend tax rate of τ^e. Equity holders face the classical double taxation of corporate income.[10]

If the dividend is negative, equity investors have to infuse capital into the firm. We apply the tax system directly to these capital infusions as well so that negative corporate earnings and dividends result in an immediate tax refund. However, we exclude capital infusions due to debt repayments from the equity investor's tax base.

In a bankruptcy node, the current firm value is split among bankruptcy claimants: debt holders, government and a bankruptcy loss. The remain-

[10]Full double taxation can be exclude by altering the equity holders tax base from dividend income.

ing value is then first distributed to debt holders proportionally to their outstanding notional. The distribution to debt holders is limited to the total notional amount. It might be possible that equity holders receive the excess portion of the residual value in bankruptcy if the bankruptcy barrier V_B allows. All bankruptcy claims except the loss portion which is excluded from tax considerations are treated like equity for tax purposes, so that the final corporate earnings and final dividend are taxed accordingly.

To improve our numerical values we implement lattices not until the end of the longest lasting debt issue but use the analytical formulas derived in Genser (2005b), Sections 3.2 and 3.3, at each terminal node of the trinomial tree.

Note that the analysis can be extended to other tax regimes easily by changing the cash flows from equity investors and the firm to the government according to the assumptions.[11].

2.3 Security Valuation

All security prices are derived from the EBIT process. Having determined all cash flows to the claimants and terminal security values, expectations in all other nodes before are discounted at the riskless interest rate. This leads to values for the total firm value, the market value of all debt issues, and equity, and finally the value of tax payments.

The same procedure can be used to price options on equity, since we have equity values at each single node. Call option values at option maturity are

$$C_T = \max(E_T - X, 0),$$

where E_T denotes the price of equity at a specific node and X the exercise price. From these terminal values, we move backwards through the tree using the risk-less interest rate and the probabilities implied by the EBIT-process in equations (3).

3 Numerical Integration Scheme

If only European derivatives are studied which depend solely on their value at option maturity and if bankruptcy of the firm knocks out the derivative, its prices can be calculate computationally more efficiently.

The value of a derivative $Y_{t_0}(\eta, T)$ as of time t_0 with maturity T can be calculated as its expected payoff at maturity under the risk-neutral

[11] See e.g. Genser (2005a) for alternative assumptions about the tax system.

probability measure.[12]

$$
\begin{aligned}
Y_{t_0}(\eta, T) &= e^{-r(T-t_0)} E^{\mathbb{Q}}_{t_0}[Y_T(\eta_T, T)] \\
&= e^{-r(T-t)} \int_{-\infty}^{\infty} Y_T(\eta_T, T)(1 - \phi_T(t_0, T, \eta_{t_0}, \eta_T, \eta_B(T))) d\eta_T, \quad (9)
\end{aligned}
$$

where $\phi_T(t_0, T, \eta_{t_0}, \eta_T, \eta_B(T)) = P(\eta_T \in d\eta, \tau > T)P(\tau > T)$ denotes the joint probability of reaching a level of η_T at the derivative's maturity and the firm going bankrupt before T when starting today at η_{t_0}. This probability is deduced in Ammann and Genser (2005) as a byproduct of the derivation of the bankruptcy probabilities . Recall that this bankruptcy probability for the interval $]t_0, t_j = T]$ is

$$
\Phi(t_0, T, \eta_{t_0}, \eta_B(T)) = 1 - P_\nu(X_{t_j} \leq z_{t_j}, M_{t_j} < y_{t_j}), \quad (10)
$$

with[13]

$$
\begin{aligned}
&P_\nu\left((-1)^{p_j(\mathcal{A}_i)} X_{t_j} \leq z_{t_j} - 2\sum_{i=1}^{j}(-1)^{p_j(\mathcal{A}_i)-1} y_{t_i} \mathbf{1}_{\{i \in \mathcal{A}_i\}}\right) \\
&\qquad = e^{z_n^* \frac{\nu}{\sigma^2}} N_n\left(\frac{z_{t_j} - z_j^* - \Omega_{j,n}\nu}{\sigma\sqrt{t_j}}, \Omega(\mathcal{A}_i)\right).
\end{aligned}
$$

Differentiating this equation with respect to z_T, results in the desired density

$$
\phi_T(t_0, T, \eta_{t_0}, \eta_T, \eta_B(T)) = \frac{\partial\Phi(t_0, T, \eta_{t_0}, \eta_T)}{\partial z_T}. \quad (11)
$$

For the special case where the bankruptcy barrier is constant or the derivative's maturity lies before the first capital restructuring, the density simplifies to[14]

$$
\phi(t_0, T, a, b) = \frac{\exp\left\{\frac{\mu a}{\sigma_\eta^2} - \frac{\mu^2 T}{2\sigma_\eta^2}\right\}}{\sigma_\eta\sqrt{T}}\left[n\left(\frac{-a}{\sigma_\eta\sqrt{T}}\right) - n\left(\frac{2b-a}{\sigma_\eta\sqrt{T}}\right)\right]. (12)
$$

In equation (12), $n(\cdot)$ denotes the standard normal density, a the starting value of the state variable and b its terminal value.

In general, the integral of equation (9) can only be solved analytically if the payoff function $Y_T(\eta, T)$ is well behaved. For option prices in the Corporate Securities Framework with several finite maturity debt issues,

[12]See e.g. Cochrane (2001).

[13]Respective definitions of variables, sets, and notational conventions are given in Genser (2005b) Subsection 3.2.3.1.

[14]See Harrison (1985).

closed form solutions cannot be derived.[15] However, the value of the derivative at maturity $Y_T(\eta_T, T)$ can be calculated analytically for each η_T since security values in the Corporate Securities Framework can be calculated explicitly. Therefore, we use numerical methods[16] to evaluate the integral in equation (9). Using the call option payoff $Y_T(\eta_T, T) = C_T$ yields the desired call option prices on the firm's equity.

Note that it is easier to differentiate numerically equation (10) because the resulting multivariate normal densities would include 2^N terms of both the multivariate normal distribution function and its density where N denotes the number of barriers. Depending on the accuracy of the approximation of the multivariate normal distribution, a numerical differentiation of hitting probabilities can build up considerable approximation errors that prevent the numerical integration algorithms for equation (9) from converging in reasonable time. To overcome the numerical problems, the hitting probability of equation (10) can be used to calculate sufficient data points to be able to spline the distribution function. It is numerically more efficient to spline the distribution function because the error accumulating numerical differentiation of the hitting probability is avoided. Moreover, the distribution function is monotonously increasing and has therefore an easier shape than the density. By differentiating the spline, equation (11) can be extracted with much higher accuracy. The probabilities can be found by evaluating the spline at the respective ending values.[17]

4 Numerical Results

In the following subsections, equity option prices in our Corporate Securities Framework are compared to the Black/Scholes framework by calculating implied Black/Scholes volatilities. Firms with EBIT following an arithmetic and a geometric Brownian motion are analyzed, we can show that the general structure of implied volatilities does not only depend on

[15] Toft and Prucyk (1997) analyze call prices on leveraged equity in a setting where only one perpetual debt issue is outstanding. This simplifies the analysis and allows the derivation of explicit formulae.

[16] Some numerical methods are sensitive to changes in the boundary values if the integrand only has values different from zero over closed interval. We therefore integrate from the bankruptcy-EBIT η_B to an upper bound of $\eta_{t_0} + 8\sigma_\eta\sqrt{T}$. Above the upper bound probabilities $\phi_T(t_0, T, \eta_{t_0}, \eta_T, \eta_B(T))$ are virtually zero and no value is added to the integral.

[17] Note that a similar method is used to extract implied densities from traded option prices. There the strike/implied volatility function is splined to extract the distribution function of equity prices at option maturity. See e.g. Brunner and Hafner (2002) and the references therein.

the distributional assumption but primarily on the firms being allowed to go bankrupt and its capital structure.

A plot of implied volatilities against strike prices is used to visualize the functional form of the option prices. A plot of the unconditional partial equity density at option maturity helps for an in-depth analysis of the implied volatility structures.[18]

A comparative static analysis is performed for the EBIT-volatility, the risk-neutral EBIT-drift, the risk-free interest rate, the EBIT-starting value, and the financing structure of the firm. The following subsections summarize and interpret the results of the simulation.

4.1 The Base Case Firm

Each firm in the economy faces a constant risk-free interest rate of $r = 5\%$, which is a widely used interest rate level in numerical examples throughout the literature. The government is assumed to tax corporate earnings at a tax rate $\tau^c = 35\%$, income from equity investments and coupon income at $\tau^d = \tau^e = 10\%$. The corporate tax rate is at the upper edge of what is observed in the European Union. The personal tax rates are chosen to reflect that smaller investors are tax exempt on their investment income in many countries or evade taxes by shifting capital abroad so that only the net effect of taxation on corporate securities' prices is modeled.

The firm specific factors that are common among GBM- and ABM-firms are its current EBIT-level of $\eta_{t_0} = 100$. Since short term debt seems to have a major impact on option prices, both firms have issued only one short term bond with a maturity of $T_1 = 1$ year, a notional of $P_1 = 1'850$, and a coupon of $C_1 = 4.5$ %, and one perpetual bond with a notional of $P_2 = 1'250$ paying a coupon of $C_2 = 6$ %.

The loss of firm value in default is set to $\alpha = 70$ % (65 %) in the ABM-case (GBM-case)[19] which yields bankruptcy barriers of $V_B(T_1) = 5'083.33$ ($4'375.14$) and $V_B(T_2) = 2'083.33$ ($1'785.71$) if 50 % of the total notional

[18] The equity density is called partial because it only starts at an equity value of zero and the intensity of the zero value is not displayed in the graphs. As a result, the shown densities need not integrate to one. The "missing" probability mass is attributable to bankruptcy.

[19] Although the bankruptcy loss ratio α appears high, consider that α is measured with respect to total firm value in case of bankruptcy and does not refer to a loss rate of market value of firm's assets. Alderson and Betker (1995) estimate the mean percentage of total values lost in liquidation to be 36.5 % of a sample of 88 firms liquidated in the period from 1982 to 1993. Gilson (1997) reports a mean percentage liquidation cost of 44.4 %. His sample contained 108 firms recontracting their debt either out-of-court or under Chapter 11 in the period 1979-1989. Our α must be higher because it refers to a firm value representing all value from future EBIT-payments in a bankruptcy node.

of debt[20] is to be recovered in case of bankruptcy.

The only missing parameters are those for the ABM- and GBM-processes, respectively. If we choose a risk-neutral ABM-EBIT-drift under the measure $\mathbb{Q}$ of $\mu = 10$ and a standard deviation of $\sigma_\eta = 40$, a GBM-process with parameters $\bar{\mu} = 3\ 1/3\ \%$ and $\bar{\sigma}_\eta = 18\ \%$ results in approximately the same security values as the ABM-case.

4.2 Comparison of Numerical Methods

The trinomial lattice approach was implemented by using 900 steps[21] until option maturity and $\lambda \approx 1.75$. Due to the short option maturity a higher step size was needed to better grasp the knock-out feature of the option. Since the first bankruptcy barrier is most important, it appeared useful to choose λ to hit the first barrier exactly. For firms close to bankruptcy, λ is set close to 1.25 to get enough non-bankrupt nodes.

For the numerical integration approach, the risk-neutral distribution function was approximated by 200 points from the bankruptcy-EBIT to 8 standard deviations above the expected EBIT at option maturity. The accuracy of the multivariate normal distribution was set to 1e-8 so that each point had a maximum accumulated error of not more than 1e-6. The spline of the equity value distribution function at option maturity has an even lower error because it tends to eliminate errors of different sign. The numerical integration is performed with an error of 1e-6.

The trinomial tree approach and the numerical integration method give identical option prices up to minor approximation errors due to the numerical methods. Table 1 summarizes the differences of option prices and implied volatilities for the base case scenario of $\eta_{t_0} = 100$ and the option maturity being 6 months. The table reports in Panel A the relative option price difference and in Panel B the relative difference of implied volatilities. We choose to report relative differences to account for level effects.

Despite the fact that option prices and implied volatilities are very sensitive to the approximation, the differences between option prices are small given that they range from -0.0179 % to 0.2584 % of the respective price of the numerical integration approach. The respective range for implied volatilities is -2.1731 % and 0.1078 % of the implied volatilities of the numerical integration approach. Especially the mean differences

[20] This is in line with usual assumptions. A standard reference is Franks and Tourus (1994) who report that on average 50.9 % of face value of total debt is recovered by debt holders.

[21] In the tree approach option prices were hard to approximate when the bankruptcy barrier changed during the option's life. If the option maturity falls after the short-term debt maturity, 1'100 steps were needed to get reasonable accuracy.

Table 1: Summary of relative differences of equity option prices (Panel A) and implied volatilities (Panel B) of the $\eta_0 = 100$, $T_O = 0.5$ scenario sets between the numerical integration and the lattice approach. The table reports the minimum and maximum relative differences as well as the mean absolute difference (MAD), the mean difference (MD), and the standard deviation of the mean difference (SDev). All numbers are in % of the values of the numerical integration approach.

Panel A: Relative Option Price Differences

Scenario	**Obs.**	**Min**	**Max**	**MAD**	**MD**	**SDev.**
All ABM and GBM scenarios	609	-0.0179	0.2584	0.0087	0.0035	0.0223
ABM only	315	-0.0151	0.1697	0.0079	0.0022	0.0182
GBM only	315	-0.0179	0.2584	0.0094	0.0048	0.0252
OTM-options	290	-0.0179	0.2584	0.0138	0.0072	0.0313
ITM-options	290	-0.0109	0.0366	0.0040	0.0002	0.0058

Panel B: Relative Implied Volatility Differences

Scenario	**Obs.**	**Min**	**Max**	**MAD**	**MD**	**SDev.**
All ABM and GBM scenarios	609	-2.1731	0.1078	0.0430	-0.0342	0.1408
ABM only	315	-0.0355	0.0559	0.0102	-0.0032	0.0133
GBM only	315	-2.1731	0.1078	0.0760	-0.0662	0.1904
OTM-options	290	-0.2532	0.0690	0.0248	-0.0175	0.0373
ITM-options	290	-2.1731	0.1078	0.0621	-0.0518	0.1985

MD are very comforting with only 0.0035 %. Comparing the maximum and the minimum pricing differences in Panel A, both approaches tend to prices in-the-money options and options of the ABM-EBIT-model equally well whereas higher price differences for the GBM-model and out-of-the-money options can be observed. However, the worst mean differences of 0.0072 % for out-of-the-money options is still remarkably good.

Looking at implied volatilities in Panel B, the pattern of relative differences is almost similar. ABM-option implied volatilities differences are again smaller than those of GBM-option implied volatilities. However, in-the-money option implied volatilities show higher differences than out-of-the-money implied volatilities. The effect is not surprising because – in contrast to out-of-the-money options – implied volatilities of in-the-money options tend to be particularly sensitive to the approximated option price and the expected equity value. However note, that the mean differences of implied volatilities are sufficiently low, as well.

4.3 Equity values and their densities at option maturity

Analyzing the densities of equity values becomes quickly difficult if the firm has a complex capital structure. To differentiate between different effects and to compare different scenarios Tables 2 and 4 summarize the first four moments of the equity distribution (Columns 9 to 12) and its return distribution (Columns 13 to 15) at option maturity[22] for all scenarios and for ABM- and GBM-firms, respectively. Panel A depicts the case of a 6 month option, Panel B the one of a 9 month option. The first three rows cover the base case firm with different initial EBITs η_0. Rows 4 and 5 illustrate the case of changes of the risk-neutral EBIT-drift μ, followed by two rows of the case of changed EBIT-volatility σ_η, risk-less interest rates r, and two rows of different option maturities T_O. The last four rows depict scenarios with different maturities for the short-term bond T_1. Figures 6 and 8 depict the equity value densities of the ABM- and GBM-firm in the 6 scenarios: a shift of the initial EBIT (Panel A), the risk-neutral drift (Panel B), the EBIT-volatility (Panel C), the risk-free interest rate (Panel D), the option maturity (Panel E), and the maturity of the short-term bond (Panel F). In the accompanying Figures 7 and 9, the density plots of equity values are translated into the corresponding equity return density plots.

4.3.1 General Comments

Equity is the residual contract to EBIT with a right to abandon future obligations. This has several effects on the risk-neutral density of equity values for a future point T_O if a good-state firm approaches bankruptcy. We start the discussion with the ABM-case.

One of the decisive elements of the moments of the equity value distribution is the position of the expected equity value. From Figure 2 we gain the insight that the expected equity value moves from the center of the distribution further towards zero if initial EBIT approaches zero. The position of the expected equity value in the partial density influences equity value moments considerably. The change of the moments of the equity value distribution is illustrated in Figure 3. As can be seen, the distribution and its moments undergo different phases.

(i) If an ABM-firm is far from bankruptcy, the density of equity is almost normal. Given a capital structure, finite maturity debt values

[22]The first four central moments shown are calculated by numerical integration of the respective expectation similar to option prices in equation (9). Without loss of generality, we define the return distribution of equity values with respect to its expected value. Therefore, the expected values of the return distribution are zero in all scenarios.

Table 2: Unconditional central moments of the equity and its return distribution at option maturity as well as LS-regression results of the form $\ln(\sigma) = \beta_0 + \beta_1 \ln\left(\frac{X}{E_t}\right)$ in the ABM-Corporate Securities Framework. The base-EBIT $\eta_0 = 100$ and parameters are changed as displayed in the first 6 columns. The bankruptcy barrier is set such that the recovery of debt holders is 50 % of total debt outstanding and losses in case of bankruptcy are $\alpha = 70$ %. T_1 denotes the maturity of the short bond, $\Phi(T_O)$ the bankruptcy probability up to option maturity T_O.

Panel A

$\eta(0)$	μ	σ_η	r	V_B	T_1	T_O	$\Phi(T_O)$	$E^{\mathbb{Q}}(E(T_O))$	$\sigma(E(T_O))$	$\zeta(E(T_O))$	$\kappa(E(T_O))$	$\sigma(r_E(T_O))$	$\zeta(r_E(T_O))$	$\kappa(r_E(T_O))$	$exp(\beta_0)$	β_1	p-value
75.0	10.00	40	5.00%	5083.33	1.00	0.500	40.2354%	640.63	453.47	1.7795	3.7381	49.79%	0.8273	4.0334	1.6562	-0.5396	0.00
100.0							7.8111%	1205.89	415.28	0.4427	2.9036	40.40%	-2.0186	11.1134	0.6396	-0.9108	0.00
125.0							0.7781%	1625.25	403.06	-0.2763	3.4487	30.39%	-2.4926	15.0766	0.3688	-0.8365	0.00
100.0	8.00						81.1339%	185.78	384.86	2.8430	8.7371	71.49%	2.4950	6.4728	3.7620	-0.2015	0.00
100.0	15.00						0.0000%	2662.99	358.18	-0.0118	3.0187	13.83%	-0.6551	3.6748	0.1934	-0.5102	0.00
100.0		30					1.8075%	1311.98	319.59	-0.3454	3.6215	30.44%	-2.7058	16.7824	0.3839	-1.0831	0.00
100.0		50					16.1224%	1101.28	487.79	1.1189	3.1794	45.03%	-1.2774	8.3216	0.9108	-0.7493	0.00
100.0			4.00%				0.0000%	2761.85	448.08	-0.0091	3.0106	16.90%	-0.8020	3.9919	0.2327	-0.5055	0.00
100.0			5.50%				92.3149%	81.74	292.62	4.2235	18.9662	69.87%	3.7564	14.3446	5.0367	-0.1265	0.00
100.0						1.000	18.5796%	1250.11	465.23	1.7271	4.0329	29.93%	1.1891	2.8677	0.6105	-0.8920	0.00
100.0						2.000	18.5988%	2230.36	768.89	1.5186	3.4094	29.37%	0.6149	4.4332	0.6818	-0.5958	0.00
100.0					0.25		1.6363%	2364.47	370.51	0.2903	3.0306	15.88%	-0.4819	3.8408	0.5898	-1.0769	0.00
100.0					0.75		7.8111%	1231.06	390.92	0.4954	3.1376	36.47%	-2.0377	12.5244	0.5971	-0.9953	0.00
100.0					1.50		7.8111%	1171.37	435.42	0.4780	2.7821	43.75%	-1.9311	9.8993	0.6810	-0.8060	0.00
100.0					3.00		7.8111%	1126.61	447.80	0.5648	2.8050	46.44%	-1.8343	9.0008	0.7166	-0.7134	0.00

Panel B

$\eta(0)$	μ	σ_η	r	V_B	T_1	T_O	$\Phi(T_O)$	$E^{\mathbb{Q}}(E(T_O))$	$\sigma(E(T_O))$	$\zeta(E(T_O))$	$\kappa(E(T_O))$	$\sigma(r_E(T_O))$	$\zeta(r_E(T_O))$	$\kappa(r_E(T_O))$	$exp(\beta_0)$	β_1	p-value
75.0	10.00	40	5.00%	5083.33	1.00	0.750	47.6579%	654.46	523.34	1.8376	3.8319	51.68%	1.3333	3.3410	1.5753	-0.4889	0.00
100.0							13.7522%	1228.50	446.94	1.0660	3.3341	37.03%	-1.3127	10.2378	0.6298	-0.9340	0.00
125.0							2.5796%	1650.77	451.70	0.1342	3.2643	32.12%	-2.1618	13.3058	0.3627	-0.8067	0.00
100.0	8.00						83.9805%	188.91	427.49	2.9499	9.3417	72.39%	2.6412	7.1579	3.2852	-0.1823	0.00
100.0	15.00						0.0006%	2705.31	438.66	-0.0135	3.0249	16.90%	-0.8294	4.3224	0.1919	-0.5079	0.00
100.0		30					4.5342%	1337.25	342.38	0.2403	3.4897	29.56%	-2.2579	15.7406	0.3760	-1.0760	0.00
100.0		50					23.9330%	1120.98	546.56	1.5499	3.7139	42.51%	-0.3310	6.6534	0.8989	-0.7486	0.00
100.0			4.00%				0.0009%	2798.91	548.75	-0.0102	3.0119	20.87%	-1.0339	4.9413	0.2312	-0.5045	0.00
100.0			5.50%				93.4312%	83.15	319.18	4.3925	20.3326	68.91%	4.0033	16.2126	4.2848	-0.1183	0.00
100.0						0.500	7.8111%	1205.89	415.28	0.4427	2.9036	40.40%	-2.0185	11.1144	0.6396	-0.9108	0.00
100.0						1.000	18.5796%	1250.11	465.23	1.7271	4.0329	29.93%	1.1891	2.8677	0.6105	-0.8920	0.00
100.0						2.000	18.5988%	2230.36	768.89	1.5186	3.4094	29.38%	0.6147	4.4440	0.6818	-0.5958	0.00
100.0					0.25		1.6366%	2394.83	467.62	-0.0612	4.0098	19.89%	-0.8657	5.1310	0.4958	-0.9898	0.00
100.0					0.75		13.7522%	1253.98	403.03	1.5582	3.8167	27.33%	0.8803	2.7616	0.5762	-0.9417	0.00
100.0					1.50		13.7522%	1193.54	480.26	0.9597	3.0557	42.37%	-1.4494	9.1353	0.6789	-0.8137	0.00
100.0					3.00		13.7522%	1148.22	501.85	0.9993	3.0741	46.01%	-1.4099	8.2204	0.7164	-0.7091	0.00

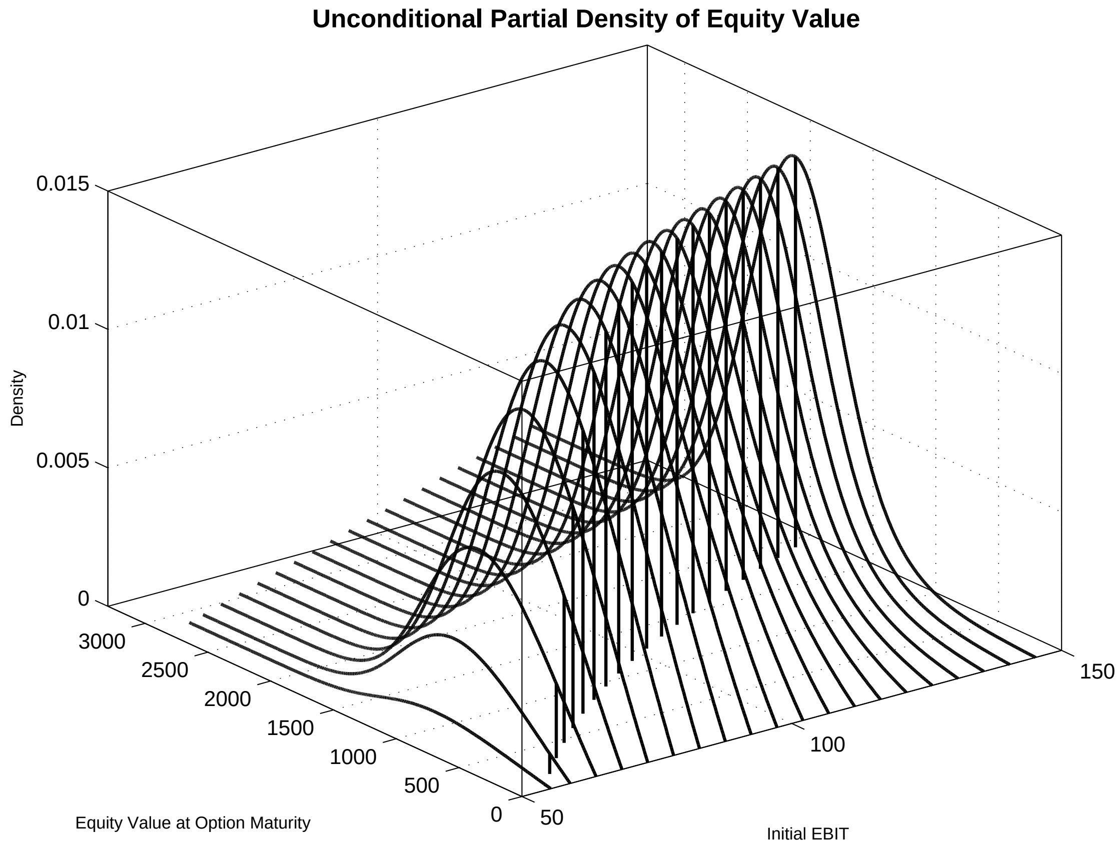

Figure 2: Equity value densities of 6 month equity options in the ABM-Corporate Securities Framework as a function of η_0. Expected equity values are indicated by solid lines. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

at option maturity are rather insensitive to changes of EBIT[23] because debt has an upper value limit, i.e. its risk-free counterpart. Equity of firms far from bankruptcy gain with each increase of EBIT a constant amount making the equity distribution symmetric and driving excess-kurtosis to zero.

Taking the high risk-neutral drift and low interest rate scenarios in

[23]The sensitivity of a security at option maturity with respect to the EBIT-level is important because we integrate over the product of the equity values which depends on the EBIT prevailing at option maturity, and the probability of occurrence which is a function of both the initial EBIT and the EBIT at option maturity, from the bankruptcy barrier to infinity.

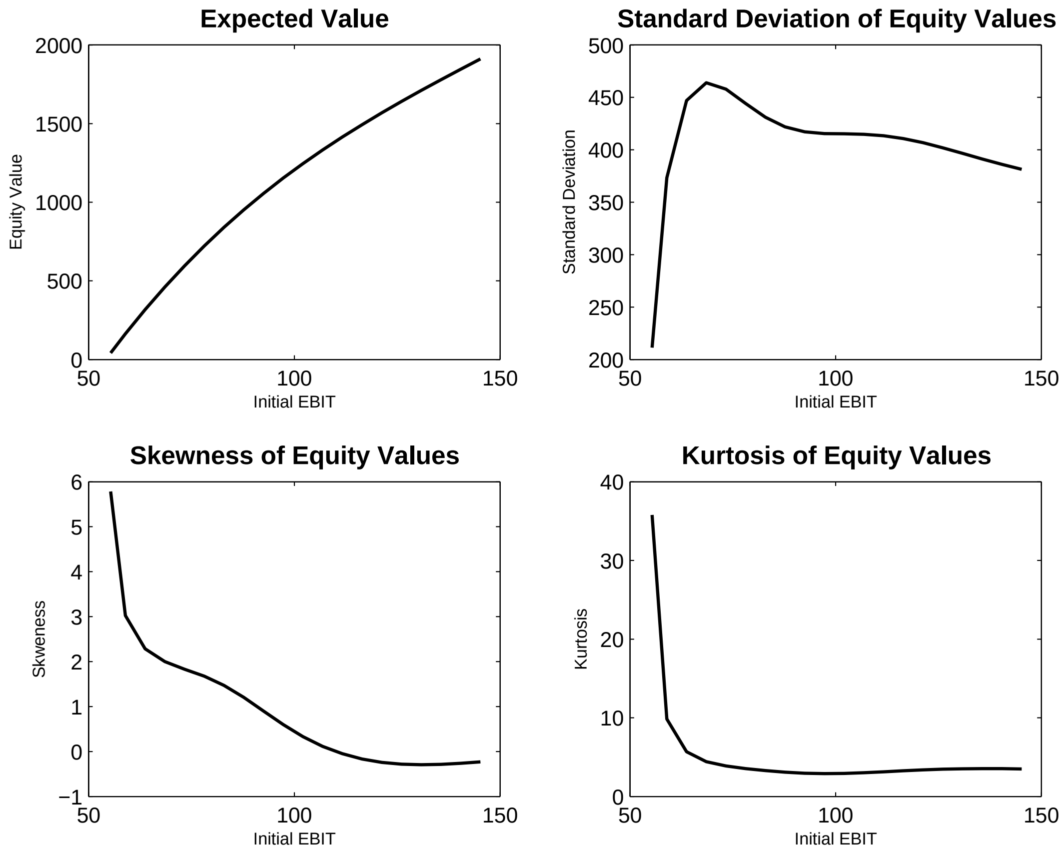

Figure 3: Equity value density moments of 6 month equity options in the ABM-Corporate Securities Framework as a function of η_0. The moments are obtained by numerical integration.

Table 2 as an example, the skewness is slightly below 0 and kurtosis slightly above 3.

(ii) The abandonment option bounds the value of equity from below at zero. The equity density will therefore exhibit a mass concentration at zero which is equal to the bankruptcy probability for a given time in the future. As a result, the continuous part of the unconditional distribution of equity value only integrates to $1 - \Phi(t_0, T, \eta_{t_0}, \eta_B(T)) \leq 1$. The bankruptcy probability pulls the expected equity value towards zero which implies that debt issues leave the region where they are insensitive to bankruptcy. However, finite maturity debt is still a sticky claim despite the slightly higher bankruptcy probability because they receive some recovery in bankruptcy. The stickiness of a debt issue depends on its maturity. Thereby, shorter maturity bonds are less sensitive than longer-term bonds. As a result, equity value suffers higher losses than debt if EBIT decreases but benefits more if EBIT increases. The asymmetry increases when approaching bankruptcy. It causes equity value skewness to decrease and excess-kurtosis to increase. The behavior of equity value skewness directly depends on the redistribution of assets for different realizations of EBIT at option maturity. If the firm moves closer to bankruptcy, equity value suffers from higher values of bankruptcy losses while finite maturity debt values are less affected. The probability of low equity values increases more than the normal distribution would predict, which skews the equity distribution to the right.[24]

This type of the equity value distribution is illustrated by the low risk and the high EBIT-value scenarios in Table 2.

(iii) If the firm moves further towards bankruptcy, the bankruptcy event starts to dominate the shape and moments of the distribution. Equity skewness increases rapidly from its intermediate lower values. Equity kurtosis decreases to values even below 3. These effects are only driven by the mass concentration of the equity value distribution at 0 and have no direct economic interpretation. Pick as an example the base case 100-EBIT-firm and the cases of the short-term debt with longer maturities (Table 2). All these scenarios have in common that they have a modest bankruptcy probability, but a relatively high standard deviation.

(iv) Close to bankruptcy, equity values are dominated by the bankruptcy event. Since the left tail of the distribution is no longer existing, equity value standard deviation decreases, skewness increases further, and kurtosis can reach levels significantly above 3. Good examples of these cases are the low 75-EBIT, the low risk-neutral drift, the high interest rate and the high risk firm (Table 2).

The risk-neutral density of continuous equity returns can be directly gained from the equity value density.[25] Figure 4 illustrates the equity

[24] Intuitively, we can argue directly within the trinomial tree: Fix the EBIT-tree's probabilities at each node. The vertical spacing of EBIT at one point in time is constant. However, the equity values at adjacent nodes are not equally spaced. The spacing of equity values relative to its current node's value increases close to the bankruptcy node because debt holders are senior claimants to the remaining total firm value. Therefore, the local relative volatility of equity increases close to bankruptcy. Economically this is intuitive since the equity value in a bankruptcy node is 0 to which a positive value of equity is pulled to.

[25] See the note on the conversion of densities in Appendix A.

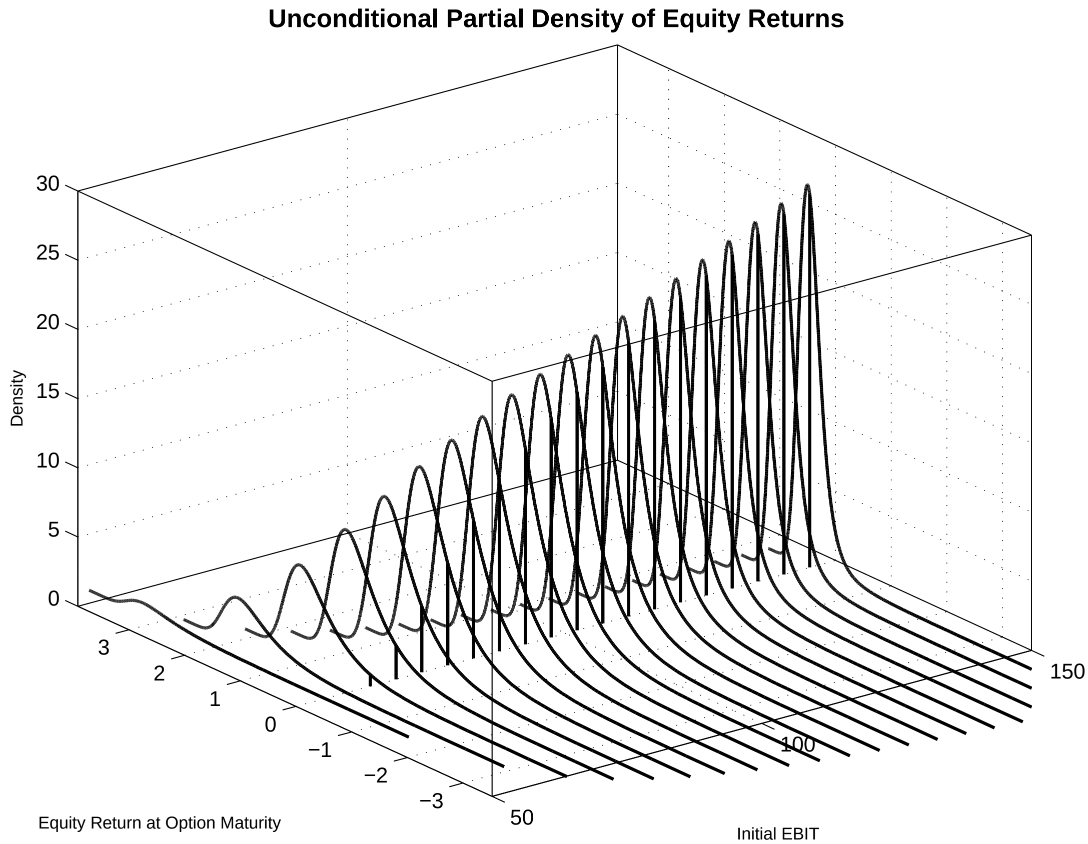

Figure 4: Equity return densities of 6 month equity options in the ABM-Corporate Securities Framework as a function of η_0. The 0-returns are indicated by solid lines. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

return densities for initial EBIT ranging from 50 to 150. The closer the firm moves towards bankruptcy, the more moves the peak of the return density into the positive quadrant although the return is defined relative to its expected value.[26] In contrast to the equity value distribution, the equity return distribution has support over the whole real line because as the equity value at some future point approaches 0, its continuous return goes to minus infinity. However, the return distribution integrates only to $1-\Phi(t_0, T, \eta_{t_0}, \eta_B(T)) \leq 1$, as well. Therefore, bad state firms (low initial EBIT, low risk-neutral drift, or high risk-less interest rate) have a positive

[26]See the zero-return line of each return density.

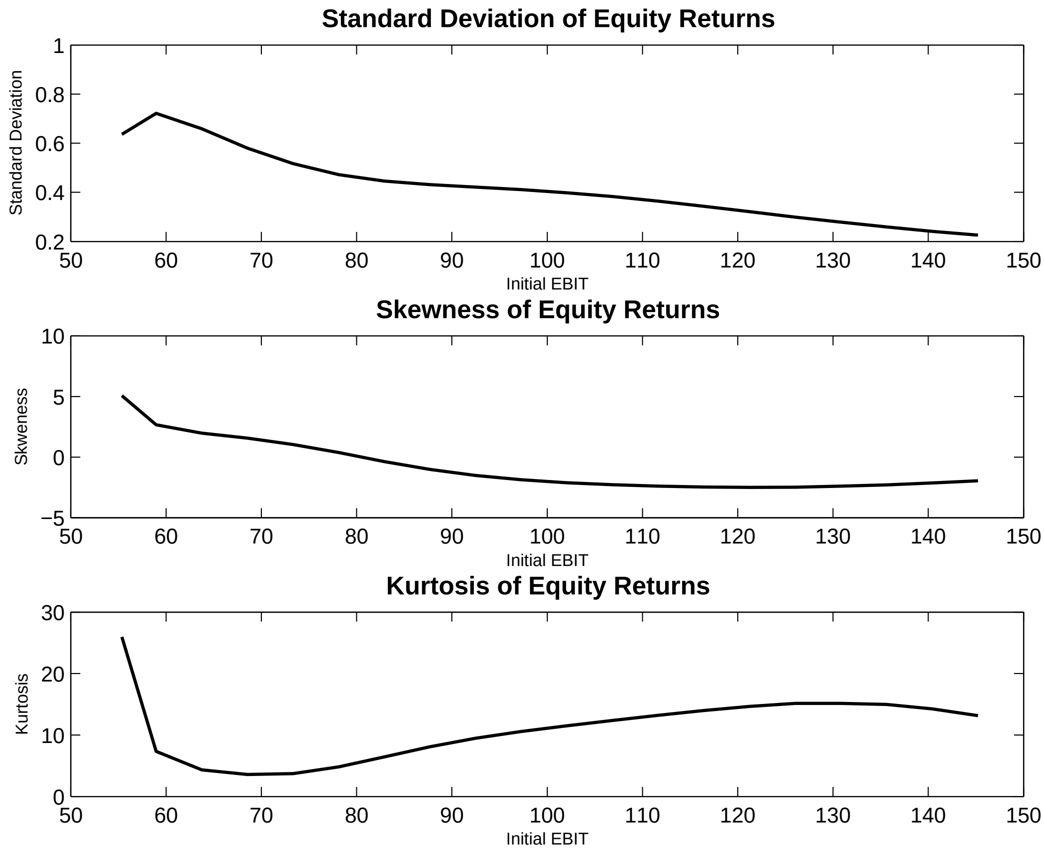

Figure 5: Equity return density moments of 6 month equity options in the ABM-Corporate Securities Framework as a function of η_0. The moments are obtained by numerical integration.

expected return given that they survive until option maturity, and a positive skewness (see also Panels B and E of Figures 7 and 9, respectively) alongside with high kurtosis. It is exactly this bankruptcy probability that complicates the interpretation of return distributions and its moments. In some cases, the unconditional moments even become quite misleading if one compares them to moments of a regular distributions.

Comparing Figures 3 and 5, the return moments follow an almost similar pattern as the equity value moments described above although at a different level and with changes at a different scale. However, some additional notes might be warranted. The equity return standard deviation depends directly on the relative level of expected equity value and its standard deviation. The equity return standard deviation is driven by two effects: (i) the equity standard deviation increases in absolute terms and (ii) the expected equity value decreases. Both effects increase return standard deviation until equity value standard deviation drops low enough to allow equity return standard deviation to fall as well. Therefore, equity return standard deviation starts to drop much closer to bankruptcy than the equity value standard deviation.

Return skewness is expected to be negative in general because a decrease of EBIT always results in a relatively larger decrease of equity value than an increase. Therefore, return skewness is much lower than equity value skewness and only the bad state firms encounter positive return skewness because of the missing left tail. By the same argument, the tails of the return density are *thicker* than normal thus return kurtosis is much higher than the equity value kurtosis for good state firms. The swings of equity return kurtosis are more pronounced. Close to bankruptcy, the increase of return skewness and kurtosis is dampened by the relatively higher return standard deviation.

The equity value distribution of the GBM-firm runs through the same four stages described for the ABM-firm.[27] There are two distinct differences, though: First, equity value skewness never decreases below 0. Second, equity value kurtosis always exceeds 3. Both moments show the same swings the closer the firm moves towards bankruptcy. In fact, the ABM- and GBM-firm are indistinguishable if they are close to bankruptcy. Note further, that the equity return distributions show much similarity so that it becomes difficult to tell which process drives EBIT if the return distribution is the only kind of information.

Equity values are distributed neither normally nor log-normally in the ABM- and GBM-case. However, all densities *look normal* for good-state firms.

To summarize the general findings of this subsection, Table 3 gives an overview of the first four central moments of the equity value and its return distribution.

4.3.2 Comparative Statics

After the general discussion about equity value and return densities and before going into details, note that parameter changes influence the equity density due to either a reduction of total firm value and/or a redistribution of a constant total firm value among different claimants. By

[27]See Figures 14 and 15 in the appendix for the equity value densities as a function of initial EBIT and its moments. Figures 16 and 17 display the respective equity return densities and moment series.

Equity Value Distribution

Firm State	(i) Excellent		(ii) Good		(iii) Medium		(iv) Bad	
EBIT follows	ABM	GBM	ABM	GBM	ABM	GBM	ABM	GBM
$E^{\mathbb{Q}}(E_T)$	very large		large		medium		small	
$\sigma(E(T))$	small		rel. small		medium		rel. large	
$\zeta(E(T))$	≈ 0	> 0	≤ 0	> 0	> 0	$>> 0$	$>> 0$	$>>> 0$
$\kappa(E(T))$	≈ 3	> 3	≤ 3	> 3	≥ 3	$>> 3$	$>> 3$	$>>> 3$

Equity Return Distribution

$\sigma(r_E(T))$	very small	small	medium	large
$\zeta(r_E(T))$	< 0	< 0	$<< 0$	> 0
$\kappa(r_E(T))$	> 3	$>> 3$	> 3 (decr.)	$>> 3$

Table 3: The first four central moments of the equity value and its return distribution depending on the current state of the firm with respect to bankruptcy and the distributional assumption. The return distribution is centered around its expected value.

equations (4) and (7), total firm value does not depend on the volatility of the EBIT-process if the risk-neutral drifts are left unchanged.[28] A change of the volatility that leads to a change of equity value results from a redistribution of firm value, i.e. a change of the probability of bankruptcy. A change of the financing structure does not change total firm value either but has effects on the distribution of value among different claimants. All other parameter changes, those of the risk-neutral drift, interest rates, and initial EBIT-value, change firm value *and* the firm's stance towards bankruptcy.

Pick first the cases where total firm value does not change. An increase of the EBIT-volatility decreases the expected equity values (see Panel C of Figure 6). The skewness of equity values changes from negative to positive. The kurtosis starts above 3 decreases to below 3 and increases again. Thus, we observe a firm that moves through the first three types of

[28]Note that the risk-neutral EBIT-process under the measure $\mathcal{Q}$ is modeled directly. If we had started with the EBIT-process under the physical measure $\mathcal{P}$, μ_η would be the drift of the physical EBIT-process and the risk premium θ_η is needed to change the probability measure to the risk-neutral measure $\mathcal{Q}$. Then, a change of σ_η alters the risk-neutral drift by means of the risk premium. Here, we use the implicit assumption that a change of the volatility induces a change of the risk premium so that the risk-neutral drift remains unaffected.

Table 4: Unconditional central moments of the equity and its return distribution at option maturity as well as LS-regression results of the form $\ln(\sigma) = \beta_0 + \beta_1 \ln\left(\frac{X}{E_t}\right)$ in the GBM-Corporate Securities Framework. The base-EBIT $\eta_0 = 100$ and parameters are changed as displayed in the first 6 columns. The bankruptcy barrier is set such that the recovery of debt holders is 50 % of total debt outstanding and losses in case of bankruptcy are $\alpha = 65$ %. T_1 denotes the maturity of the short bond, $\Phi(T_O)$ the bankruptcy probability up to option maturity T_O.

Panel A

$\eta(0)$	μ	σ_η	r	V_B	T_1	T_O	$\Phi(T_O)$	$E^{\mathbb{Q}}(E(T_O))$	$\sigma(E(T_O))$	$\zeta(E(T_O))$	$\kappa(E(T_O))$	$\sigma(r_E(T_O))$	$\zeta(r_E(T_O))$	$\kappa(r_E(T_O))$	$exp(\beta_0)$	β_1	p-value
75.0	3.33%	18%	5.00%	4357.14	1.00	0.500	78.6057%	144.36	290.58	3.1360	12.8248	70.22%	2.4493	6.4458	3.5985	-0.2102	0.00
100.0							1.0069%	1323.58	504.69	0.4842	4.4591	45.73%	-1.8564	8.4684	0.5634	-0.4179	0.00
125.0							0.0015%	2312.42	623.62	0.4734	5.1688	28.57%	-0.9775	4.6390	0.3826	-0.2763	0.00
100.0	3.00%						26.3453%	580.39	376.12	1.9185	6.5375	53.45%	-0.4574	5.9861	1.3027	-0.5139	0.00
100.0	3.67%						0.0014%	2323.50	624.04	0.4731	5.1476	28.43%	-0.9680	4.6012	0.3813	-0.2751	0.00
100.0		13%					0.0310%	1364.97	361.03	0.2650	3.8795	29.02%	-1.3621	7.2263	0.3786	-0.3665	0.00
100.0		23%					4.7142%	1272.28	617.97	1.0015	6.6668	56.75%	-1.7032	7.1600	0.7669	-0.4233	0.00
100.0			4.00%				0.0000%	6789.70	1244.04	0.8086	10.6568	18.33%	-0.3932	3.4415	0.2547	-0.1192	0.00
100.0			5.50%				63.1049%	285.36	371.07	2.4781	7.8316	63.15%	1.8390	4.4066	2.6452	-0.3150	0.00
100.0						1.000	6.3537%	1375.84	659.58	1.1645	4.3383	48.69%	-0.8721	3.6766	0.5582	-0.4000	0.00
100.0						2.000	6.3623%	2511.92	1021.12	1.2887	4.9195	39.57%	-0.6929	4.7161	0.4763	-0.6091	0.00
100.0					0.25		0.0319%	2405.18	502.35	0.3630	3.2449	21.39%	-0.6037	3.4751	0.3716	-0.6644	0.00
100.0					0.75		1.0069%	1324.15	497.10	0.4528	3.3109	44.28%	-1.7827	8.2907	0.5556	-0.3955	0.00
100.0					1.50		1.0069%	1321.51	512.43	0.3935	3.2055	47.09%	-1.8875	8.3964	0.5757	-0.4317	0.00
100.0					3.00		1.0069%	1321.20	523.53	0.3973	3.1576	48.32%	-1.8775	8.1756	0.5892	-0.4262	0.00

Panel B

$\eta(0)$	μ	σ_η	r	V_B	T_1	T_O	$\Phi(T_O)$	$E^{\mathbb{Q}}(E(T_O))$	$\sigma(E(T_O))$	$\zeta(E(T_O))$	$\kappa(E(T_O))$	$\sigma(r_E(T_O))$	$\zeta(r_E(T_O))$	$\kappa(r_E(T_O))$	$exp(\beta_0)$	β_1	p-value
75.0	3.33%	18%	5.00%	4357.14	1.00	0.750	82.1484%	148.58	336.05	3.4339	17.1215	71.26%	2.6329	7.3133	3.1836	-0.1868	0.00
100.0							3.3795%	1349.92	594.06	0.8910	6.0670	50.56%	-1.6599	7.4386	0.5603	-0.4092	0.00
125.0							0.0370%	2347.46	770.62	0.6018	6.2762	35.93%	-1.2563	5.7652	0.3819	-0.2762	0.00
100.0	3.00%						35.5460%	593.90	447.57	2.2563	8.8675	52.62%	0.4202	5.0979	1.2833	-0.4877	0.00
100.0	3.67%						0.0349%	2362.69	771.83	0.6005	6.2314	35.69%	-1.2421	5.7264	0.3805	-0.2749	0.00
100.0		13%					0.2768%	1392.10	441.14	0.4041	4.1405	35.62%	-1.5152	7.5855	0.3766	-0.3580	0.00
100.0		23%					10.3782%	1297.22	719.96	1.6434	12.5294	57.68%	-1.3839	6.5952	0.7651	-0.4225	0.00
100.0			4.00%				0.0000%	6867.45	1536.65	0.9031	12.2042	22.53%	-0.5064	3.3914	0.2549	-0.1352	0.00
100.0			5.50%				68.9919%	292.03	433.74	2.6543	9.8096	65.84%	2.0388	4.7540	2.4232	-0.2772	0.00
100.0						0.500	1.0069%	1323.43	503.60	0.4160	3.2699	45.69%	-1.8622	8.4936	0.5637	-0.4189	0.00
100.0						1.000	6.3537%	1375.84	659.58	1.1645	4.3383	48.69%	-0.8721	3.6766	0.5582	-0.4000	0.00
100.0						2.000	6.3623%	2511.92	1021.12	1.2888	4.9192	39.56%	-0.6937	4.7278	0.4763	-0.6090	0.00
100.0					0.25		0.0319%	2437.30	625.27	0.6554	7.6604	26.50%	-0.7502	3.7543	0.3425	-0.2048	0.00
100.0					0.75		3.3795%	1350.65	585.25	0.9715	6.2502	46.16%	-1.0368	3.8813	0.5528	-0.3836	0.00
100.0					1.50		3.3795%	1347.98	605.72	0.8322	5.7932	53.05%	-1.7603	7.5811	0.5733	-0.4328	0.00
100.0					3.00		3.3795%	1347.66	620.26	0.8105	5.5245	54.87%	-1.7673	7.3857	0.5883	-0.4292	0.00

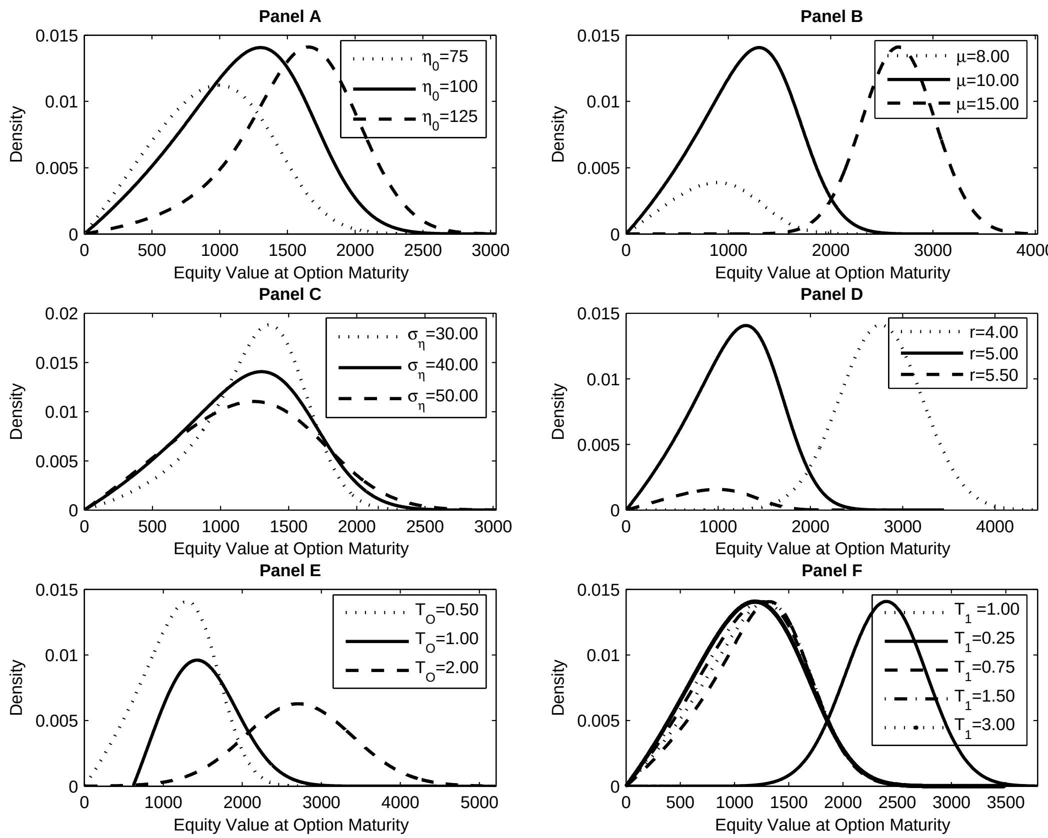

Figure 6: Unconditional partial densities of equity in the ABM-Corporate Securities Framework with $\eta_0 = 100$ at $T_O = 0.5$: Parameter changes are indicated in the legend. The bankruptcy barrier V_B is set so that 50 % of the outstanding notional is recovered in bankruptcy and bankruptcy losses are $\alpha = 70$ %. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

equity densities described above. Skewness and kurtosis change according to the discussion for the type (ii) firm.

A change of the financing structure extends short-term bond maturity and reduces the current and the expected equity value. For the ABM-case, Panel F of Figure 6 shows that financing structures with $T_1 > 0.5$ result in overlapping equity densities. In the $T_1 = 0.25$-case, the expected equity value shifts considerably to the right. Four effects drive this result: First, the 3-month bond is repaid before option maturity. Equity holders

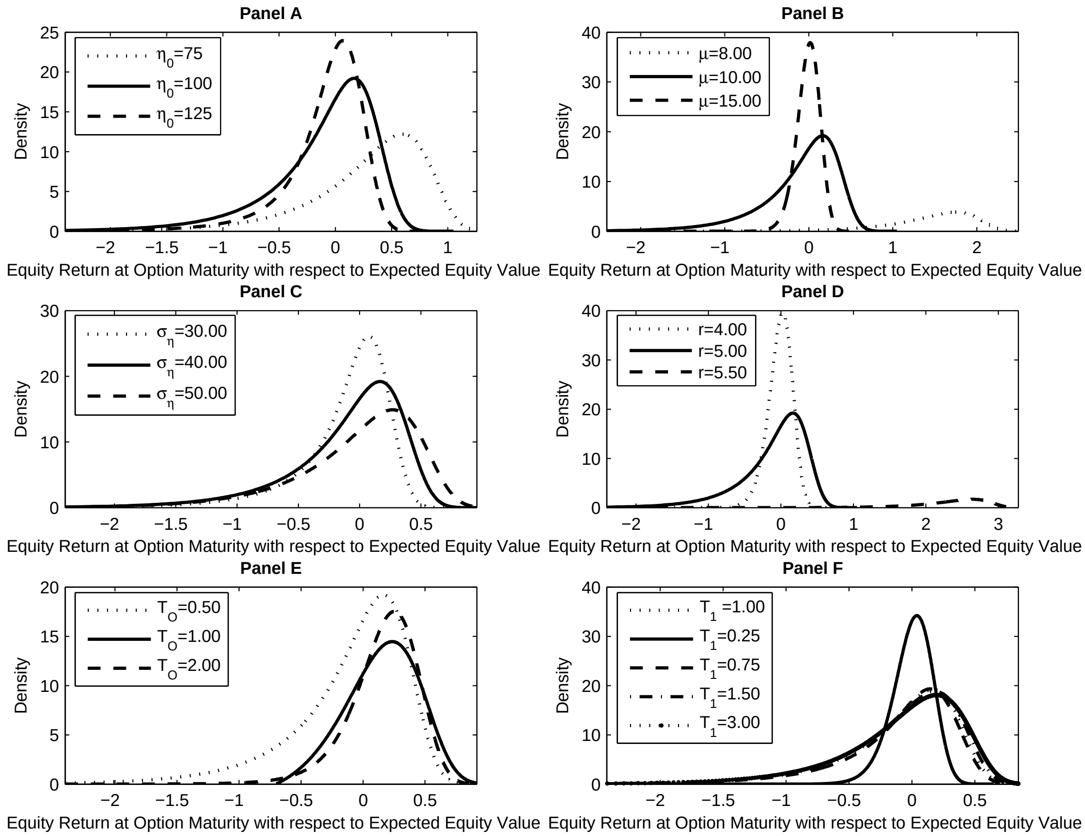

Figure 7: Unconditional partial return densities of equity in the ABM-Corporate Securities Framework with $\eta_0 = 100$ at $T_O = 0.5$: Parameter changes are indicated in the legend. The bankruptcy barrier V_B is set so that 50 % of the outstanding notional is recovered in bankruptcy and bankruptcy losses are $\alpha = 70$ %. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

do not face this payment at option maturity and so expected equity value rises. Second, due to the lower total debt outstanding the firm faces a lower bankruptcy barrier at option maturity than at t_0. If the debt maturity is increased for more than the option maturity, not only option holders have a higher probability of being knocked out[29] but also equity

[29]Note that the probability of going bankrupt in the first three months is only 1.4385 %, whereas the bankruptcy probability until option maturity with longer lasting bonds is 7.8111 %. See Table 2.

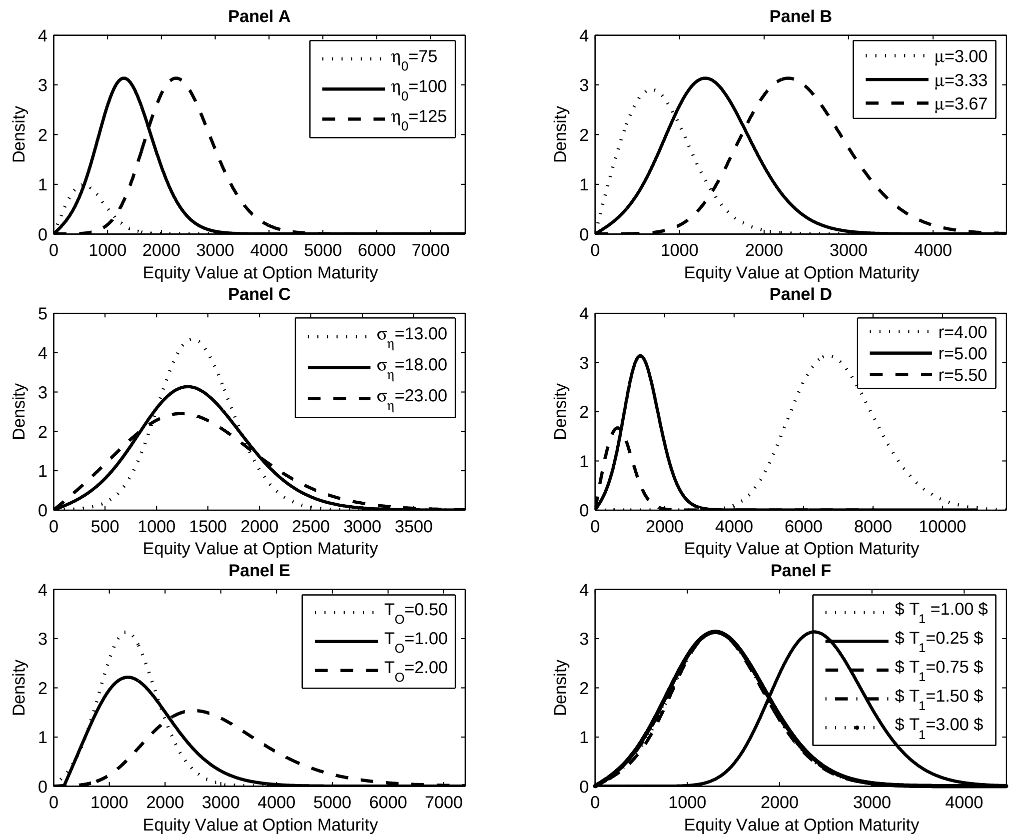

Figure 8: Unconditional partial densities of equity in the GBM-Corporate Securities Framework with $\eta_0 = 100$ at $T_O = 0.5$: Parameter changes are indicated in the legend. The bankruptcy barrier V_B is set so that 50 % of the outstanding notional is recovered in bankruptcy and bankruptcy losses are $\alpha = 65$ %. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

holders at option maturity face the higher bankruptcy barrier. Third, longer-maturity bonds pay coupons for a longer period, thus increasing debt value and decreasing equity value. Fourth, higher coupon payments imply higher tax savings due to the tax advantage of debt which shifts value from the government to equity holders. The decreasing expected equity values in Table 2 demonstrate that the coupon and bankruptcy probability effect dominate when the short-term bond maturity is increased.

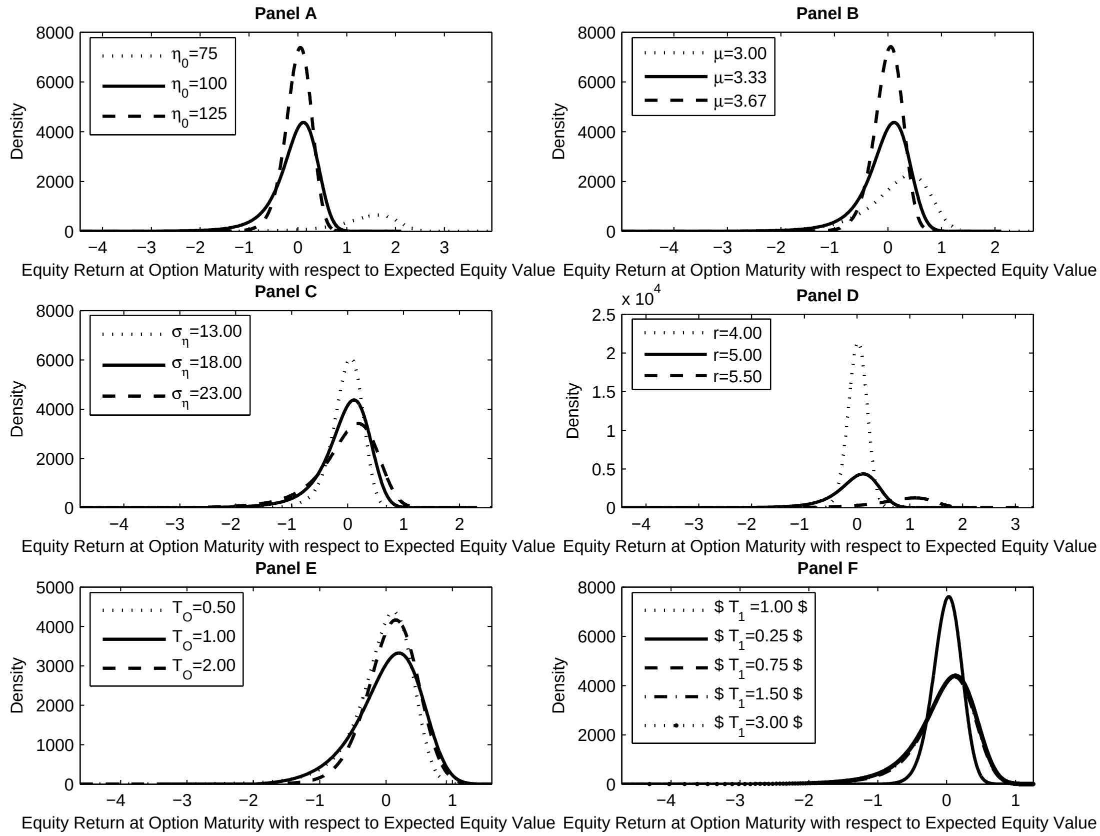

Figure 9: Unconditional partial return densities of equity in the GBM-Corporate Securities Framework with $\eta_0 = 100$ at $T_O = 0.5$: Parameter changes are indicated in the legend. The bankruptcy barrier V_B is set so that 50 % of the outstanding notional is recovered in bankruptcy and bankruptcy losses are $\alpha = 65$ %. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

The equity return standard deviations reflect exactly that behavior (Column 13 Table 2). The longer the short-term bond maturity, the higher becomes the return standard deviation. The higher moments of the equity and its return distribution are driven by the lower sensitivity of short-term bonds to changes in EBIT and the higher bankruptcy probability during the life of the short-term bond. Panel F of Figure 6 clearly displays that the bankruptcy effect dominates. For a given density value, equity values are lower in the increasing part of the equity densities the longer the maturity of the short-term bond. As a result, the equity value standard deviation and skewness increase whereas kurtosis decreases. The return moments show the same pattern. However, returns are skewed to the left and not to the right, as expected.

Table 4 and Panel F of Figures 8 and 9 depict the GBM-case which show the same effects.

Changing the option maturity T_O, i.e. the point in time at which we investigate the equity densities in the future, gives further insights into the dependence of the equity distribution on option maturity and the capital structure. Table 5 summarizes the moments of the ABM-equity value and its return distribution for maturities ranging from 3 months to 3 years where the standard deviations have been annualized by the $\sqrt{T}$-rule (see also Rows 10 and 11 of Table 2 and Panel E of Figures 6 and 7 for the respective graphs of the equity value and its return densities).

Table 5: ABM-equity value and its return moments of the equity distribution with maturities T_O from 0.25 to 3. Standard deviations are annualized by the $\sqrt{T}$-rule.

T_O	$E^{\mathbb{Q}}(E(T_O))$	$\sigma(E(T_O))$	$\zeta(E(T_O))$	$\kappa(E(T_O))$	$\sigma(r_E(T_O))$	$\zeta(r_E(T_O))$	$\kappa(r_E(T_O))$
0.25	1,182.12	702.77	-0.2432	2.9625	76.2450 %	-2.4349	12.5044
0.50	1,205.89	587.29	0.4427	2.9036	57.1358 %	-2.0185	11.1144
0.75	1,228.50	516.08	1.0661	3.3341	42.7505 %	-1.3139	10.2492
1.00	1,250.11	465.23	1.7271	4.0329	29.9259 %	1.1891	2.8677
1.50	2,179.92	557.87	1.6068	3.3781	21.2804 %	1.1930	2.9633
2.00	2,230.36	543.69	1.5187	3.4095	20.7674 %	0.6154	4.4277
3.00	2,331.06	525.84	1.4349	3.4630	19.9796 %	-0.1092	6.3654

The repayment of the short-term bond at $T_1 = 1$ has a large impact on the equity distribution. Expected equity value surges because the debt burden is reduced. Although standard deviations increase in absolute terms, the annualized standard deviations decreases but jumps up after the repayment date. This discontinuity is driven by the prevailing higher equity value and the fact that the left tail of the equity value distribution is lengthened i.e. the distance of the expected value to the bankruptcy level is extended. The return standard deviation decreases monotonically meaning that the relative riskiness of the firm decreases the longer it survives. In any case, scaling standard deviations of distributions at different points in time by the $\sqrt{T}$-rule is impossible.

The higher moments of the ABM-equity value distribution are clearly influenced by the proximity to bankruptcy. Skewness increases at first because the left tail of the distribution continues to be cut and probability mass is concentrated at 0 as bankruptcy becomes more probable. After the debt repayment, the bankruptcy barrier falls and the restitution of part of the left tail reduces skewness again. Kurtosis increases until the debt repayment date, jumps down at that date, and starts growing again. The peculiar behavior of the higher moments of the return distribution is experienced here as well. The last two columns of Table 5 exhibit a good example. The sudden jump of the bankruptcy level causes return skewness to change its sign and return kurtosis to drop below 3. Inspection of Panel E of Figure 7 does not reveal these facts!

Table 6: GBM-equity value and its return moments of the equity distribution with maturities T_O from 0.25 to 3. Standard deviations are annualized by the $\sqrt{T}$-rule.

T_O	$E^{\mathbb{Q}}(E(T_O))$	$\sigma(E(T_O))$	$\zeta(E(T_O))$	$\kappa(E(T_O))$	$\sigma(r_E(T_O))$	$\zeta(r_E(T_O))$	$\kappa(r_E(T_O))$
0.25	1,297.07	728.76	0.1512	3.1387	63.5196 %	-1.5634	7.8315
0.50	1,323.43	712.20	0.4160	3.2699	64.6123 %	-1.8622	8.4936
0.75	1,349.74	684.04	0.7829	3.7046	58.3691 %	-1.6617	7.4388
1.00	1,375.84	659.58	1.1645	4.3383	48.6858 %	-0.8721	3.6766
1.50	2,449.05	700.41	1.2914	4.6809	26.8249 %	-0.2256	3.5146
2.00	2,511.92	722.04	1.2887	4.9194	27.9808 %	-0.6931	4.7179
3.00	2,640.78	756.49	1.3994	5.5727	29.2571 %	-1.1809	5.9715

Interpretation of the moments of the GBM-equity value and return distribution as a function of maturity can follow along the same lines as the ABM-case (see Table 6, Rows 10 and 11 of Table 2, and Panel E of Figures 6 and 7). The same decrease of equity value standard deviation at the debt repayment date can be experienced. Thereafter, annualized equity value standard deviations increase slightly as expected if EBIT follows a log-normal distribution. However, equity value skewness and kurtosis increases continuously with maturity. The equity return distribution shows a more interesting pattern. In contrast to the ABM-case, equity return standard deviation increases after debt repayment. However, higher return moments show an ambiguous pattern but skewness stays negative and kurtosis above 3 for all maturities.[30]

All other changes to the base case parameter set cause a change of total firm value per se which forces a redistribution of claim values. A reduction of the risk-neutral EBIT-drift, of the initial EBIT-level, and an increase of the risk-less interest rate results in a reduction of firm

[30] Ait-Sahalia and Lo (1998) report in their Figure 7 moments of risk-neutral return distributions estimated non-parametrically from time series of option prices. Higher return moments of the S&P-500 index are instable with respect to maturity and show kinks. Effects that our simple example exhibits, as well.

value and thus of the expected value of equity. The closer the firm moves towards bankruptcy, the higher equity value skewness, the higher the kurtosis, i.e. the firms are of type (iii) and (iv). Moves in the opposite direction makes the equity densities look *normal*, i.e. firms approach those of type (i). Panels B and D of Figure 6 for the ABM-case and Figure 8 for the GBM-case illustrate this truncation of the density on the left at bankruptcy.

Comparing our return moment pattern to those reported in Table 6 of Bakshi et al. (2003), the picture fits into our simulated moments: (i) Our return distribution exhibits positive return skewness if the firm is close to bankruptcy. In Bakshi et al. (2003), IBM is not a convincing candidate for this. However, ABM-equity return densities around debt repayments dates have higher skewness as well. (ii) Our return kurtosis is generally above 3. The exception again is the ABM-equity return density around debt repayment. American International, Hewlett Packard, and IBM have average kurtosis below 3. (iii) There is a tendency that an increase of skewness implies higher kurtosis, which Bakshi et al. (2003)'s Table 6 shows, as well. Exceptions in our model are only firms just before bankruptcy, which usually have no options traded on their equity. (iv) Our volatility and skewness is generally higher.

As a final remark, differences might be due to the replication procedure used by Bakshi et al. (2003) who need to average across option maturities to get a maturity-consistent time series of option prices. Since we can resort to the whole unconditional distribution of equity at maturity, Bakshi et al. (2003)'s results might be biased within our framework and therefore be not the best comparison. Additionally, we showed above that debt repayments before option maturity have a huge impact on the equity return distribution which might distort the results of Bakshi et al. (2003). However, it is the only study so far that analyzes individual stock option.

4.4 Equity option prices and implied Black/Scholes volatilities

In option markets it is observed that option's implied Black/Scholes volatility as a function of strike prices is monotonously falling. This specific functional form is usually referred to as the option's volatility smile. The economic literature did not present an easy explanation for this phenomenon but tried technical extensions such as stochastic volatility models which produced observed volatility smiles.[31]

[31] See Jackwerth (1999) for a literature overview. Stochastic volatility models have been studied before by e.g. Heston and Nandi (2000) and Heston (1993). Dumas, Fleming and

In the Corporate Securities Framework, an inversion of the Black and Scholes (1973) formula is not possible because the prices of all security values depend only on the size of EBIT and the capital structure in a particular state at option maturity. Payments during the life of the option to debt and equity holders are irrelevant. Therefore, we can calculate equity option prices by numerical integration and by finite difference methods. The implied volatilities of the option can be backed out from the more general form of the Black and Scholes (1973) option pricing formula which is based on the expected future value of the underlying asset.[32]

$$\begin{aligned} C_{t_0} &= e^{-r(T_O - t_0)} E^{\Omega}\left[\max(E_{T_O} - X, 0)\right] \\ &= e^{-r(T_O - t_0)} \left[E^{\Omega}\left[E_{T_O}\right] N(d_1) - X N(d_2) \right] \end{aligned} \tag{13}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{E^{\Omega}\left[E_{T_O}\right]}{X}\right) + \frac{\sigma_{IV}^2}{2}(T_O - t_0)}{\sigma_{IV}\sqrt{T_O - t_0}} \\ d_2 &= d_1 - \sigma_{IV}\sqrt{T_O - t_0}, \end{aligned}$$

and σ_{IV} is the annual implied volatility of the logarithm of the underlying equity value E over the period $T_O - t_0$.

When using implied Black/Scholes volatilities as a benchmark, we compare each of our scenarios with the log-normal density of equity which Black and Scholes (1973) assumed for the underlying. If the Black and Scholes (1973) model were correct, the implied volatilities for all options must be equal.

From equation (13) it follows that high option prices imply high implied volatilities. If implied volatilities increase for lower strikes, the equity densities have more probability mass left of the strike price than the log-normal density. To see this, take a state contingent claim which pays 1 currency unit if the equity price has a certain level at maturity. As Breeden and Litzenberger (1978) point out, the price difference of two of such claims with different strike levels are related to the difference of the probability of the two states actually occurring. If the claim with the lower strike is worth more than the other as implied by the log-normal assumption implicit in the Black/Scholes model, this probability must be

Whaley (1998) provide evidence that an implied volatility tree in the sense of Rubinstein (1994) is not superior to the ad-hoc applied implied volatility curve in a Black/Scholes model. See also Grünbichler and Longstaff (1996) who analyze volatility derivatives where the volatility itself is assumed to follow a mean-reverting process. Fleming, Ostdiek and Whaley (1995) describe the properties of the quoted volatility index on the S&P 100.

[32] See, e.g. Hull (2000), p.268 ff.

higher. Thus, we can conclude that the probability mass between the strikes of these two claims is higher than implied by the Black/Scholes distributional assumption.[33]

Bakshi et al. (2003) analyze the connection between the physical equity return distribution, its risk-neutral counterpart, and the resulting equity option implied Black/Scholes volatilities on a single-stock basis. The analytical and empirical results strongly support the hypothesis that the more the risk-neutral return distribution is left-skewed, the higher the curvature of implied volatility smile. Furthermore, a higher kurtosis flattens the smile somewhat. Their Table 5 summarizes these results.

The simulation results of equity option prices in our ABM-Corporate Securities Framework support that behavior. The GBM-firm exhibits exceptions to their rule if the bankruptcy probability rises very quickly. However, we are able to give intuition to these findings. In our model, the equity return distribution results from the specification of the equity contract as being the residual claim to EBIT, a complex capital structure, and the distinct ability of equity owners to declare bankruptcy. This is the primary economic interpretation of the results found by Bakshi et al. (2003), and so completes their analysis on the economic level.

In all our examples, we find a downward sloping implied volatility curve. The level of implied volatilities and the curvature of the strike/volatility function depend on the current state of the firm towards bankruptcy expressed by the equity return standard deviation. If we detail the analysis by comparing our parameter settings, we find several stylized facts: (i) Implied volatilities of the GBM-case are higher than those of the comparable ABM-case if the firm is far from bankruptcy. The opposite is true close to bankruptcy because the GBM-volatility decreases with the EBIT-level thus changing the term structure of bankruptcy probabilities. (ii) The closer the firm is to bankruptcy, the higher the implied volatility of at-the-money options. (iii) The ABM-implied volatility structure gets flatter for higher at-the-money implied volatility levels. These firms have a high return distribution skewness and kurtosis. (iv) The GBM-firm exhibits steeper slopes at higher implied volatility levels.

It seems important to note that the linking of the level and shape of the volatility smile to equity return moments seems only suitable for good state firms. Following our discussion about higher moments of the return distribution, the bankruptcy probability might bias the relation detected by Bakshi et al. (2003). Their sample firms can be considered as being far from bankruptcy.

Figures 10, 11 display implied volatility curves against moneyness for ABM- and GBM-firms for 6 month option maturity. Hereby, moneyness

[33]See Appendix B for a more formal exposition.

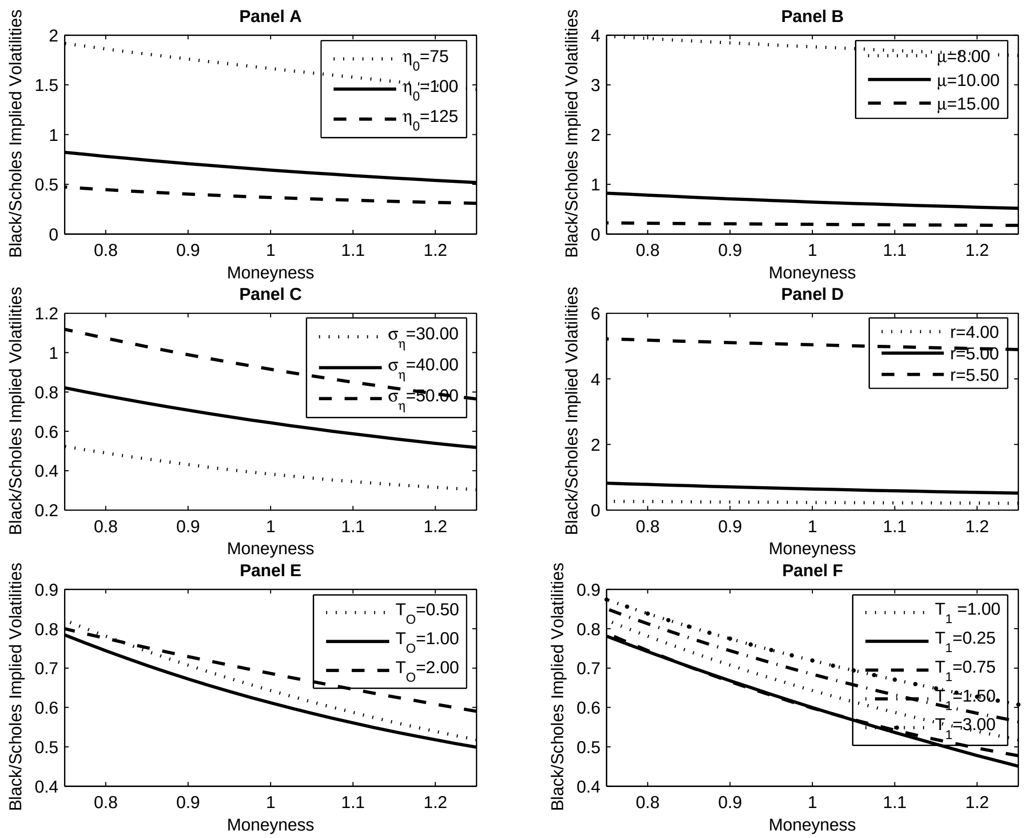

Figure 10: Implied Black/Scholes volatilities of 6 month equity options in the ABM-Corporate Securities Framework with $\eta_0 = 100$: Parameter changes are indicated in the legend. The bankruptcy barrier V_B is set so that 50 % of the outstanding notional is recovered in bankruptcy and bankruptcy losses are $\alpha = 70$ %. Option prices are obtained by numerical integration.

is defined as the fraction of the strike to the current equity value. As expected, lower EBIT-starting values (Panels A), higher interest rates (Panels D), lower risk-neutral drifts (Panels B), and higher EBIT-volatility (Panels C) take the firm closer to bankruptcy and thus show higher implied volatility levels.

The maturity of the option (Panels E) has only a major impact on the volatility smile if debt is repaid during the option's life. Then, the implied volatility smile becomes much flatter in the ABM-case and drops in the GBM-case. Recall from the last subsection that expected equity values increase due to the capital infusion by equity owners to repay debt.

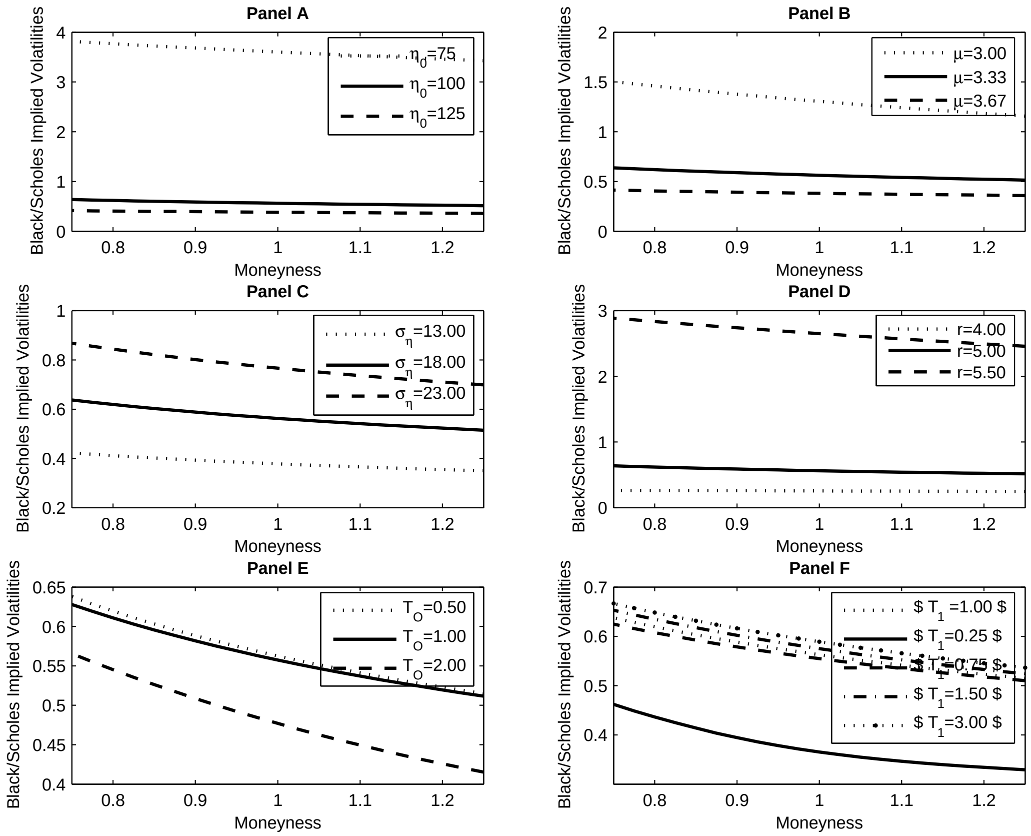

Figure 11: Implied Black/Scholes volatilities of 6 month equity options in the GBM-Corporate Securities Framework with $\eta_0 = 100$: Parameter changes are indicated in the legend. The bankruptcy barrier V_B is set so that 50 % of the outstanding notional is recovered in bankruptcy and bankruptcy losses are $\alpha = 65$ %. Option prices are obtained by numerical differentiation.

The last three columns of Tables 2 and 4 are devoted to a regression analysis as performed by Bakshi et al. (2003) in their Table 3. The implied volatility is represented by the regression model

$$\ln(\sigma_{IV}) \quad = \quad \beta_0 + \beta_1 \ln\left(\frac{X}{E_0}\right). \tag{14}$$

In equation (14), $\exp(\beta_0)$ can be interpreted as the at-the-money implied volatility. β_1 is a measure of the steepness of the implied volatility curve. For example, the figures of Rows 10 and 11 confirm that increasing the option maturity continuously increases the ATM-implied volatility, but the 2-year option has the lowest slope.

Note that the at-the-money implied volatility is always higher than the standard deviation of the equity return density for good state firms. This reflects the fact that the expected future equity value lies above the current value of equity and so the at-the-money implied volatility with respect to the expected future equity value is lower.

For firms closer to bankruptcy implied volatilities can become large. At-the-money levels of 100 % and more are common.

The effects of changes of the financing structure need a more detailed discussion. As mentioned in the last subsection, equity return standard deviations depend on the schedule of debt maturities. Recall that the earlier the short-term debt matures, the more imminent becomes debt repayment. If the firm survives, the bankruptcy barrier is lowered and the equity value jumps upwards. Extending short-term debt maturity increases the period of coupon payments and the bankruptcy barrier in the extension period which decreases current and future equity value. As a result, equity return standard deviation and ATM-implied volatilities increase. As can be seen from Figure 10 Panel F, the slope of the implied volatility smile flattens. This graph also illustrates Toft and Prucyk (1997)'s debt covenant effect although they interpret a higher covenant, i.e. a higher bankruptcy barrier, as a substitute of the amount of short-term debt. Effectively, we increase the bankruptcy barrier as well. However, we have the additional effect of a reduced (expected) equity value because the maturity of short-term debt is extended. As is demonstrated here, the term structure of the corporate capital structure matters. The simple argument of a debt covenant is not enough to explain the richness of implied volatility smiles. Again, our framework gives a very intuitive and simple explanation.

5 Concluding Remarks

In this study, we use a numerical implementation of the Corporate Securities Framework to price options on equity in order to explain the existence and the shape of volatility smiles. We compare our option prices to the Black/Scholes setting by calculating implied volatilities.

Since our distributional assumptions are different to those of the Black/Scholes environment, differences in the shapes of implied volatility curves have to be expected. However, it is interesting that our economically intuitive environment is sufficient to explain a behavior that needed much more elaborate mathematical techniques before. We find that the distributional assumption for EBIT is not crucial to the general downward sloping shape of implied volatilities but the features of the equity contract. Equity holders can stop infusing capital into the firm and declare bankruptcy. The option on equity forgoes. As a result, the unconditional distribution of equity values at option maturity is skewed and exhibits excess kurtosis. ABM-firms and GBM-firms run through several stages of distribution types. We categorized the stages by four generic types of distributions when a firm in an excellent condition moves towards bankruptcy. Equity return distributions of ABM- and GBM-firms behave surprisingly similar except for the level and the sensitivity. Good state firms exhibit small return standard deviation, a slightly negative skewness and a small excess kurtosis. If the firm moves towards bankruptcy the return standard deviation and kurtosis increase, skewness decreases. Close to bankruptcy the skewness turns positive at high levels of the standard deviation and kurtosis. The effects are due to the concentration of the probability mass of the equity value distribution at zero. As a result, moments of equity value and return distributions of firms close to bankruptcy must be interpreted carefully because the levels of higher moments might be misleading when compared to moments of regular distributions.

The effects on option prices are as follows: All curves of implied volatilities as a function of moneyness are convex and monotonously decreasing. Options on equity of firms close to bankruptcy generally show very high at-the-money implied volatilities with a decent slope. The further the firm's distance to default, the lower implied volatility levels and the steeper the slope. Financing decision can have a significant impact on implied volatilities. Repayment of debt around option maturities incurs sharp increases in implied volatilities due to the jumps by equity values. Switching to longer debt maturities might decrease equity values due to higher coupon payments which outweighs the present value effect of no immediate debt repayment. For good state firms, debt repayment can effectively decrease implied volatilities of longer lasting equity options because (i) expected equity value is increased (ii) and bankruptcy becomes less imminent once the debt burden is lowered.

In the GBM-case, the term structure of bankruptcy probabilities can influence equity values considerably and thus effectively increase the slope of implied volatilities the higher its level.

Our explanation of equity smirks is simply and intuitively linked to the economic condition of the firm and to its debt structure. We argue this is a significant progress to other studies of option smirks which use mathematically more elaborated but economically less intuitive concepts.

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A A Note on the Change in Variable of Equity Value and its Return Density Plots

The discussion in 4.3 relies heavily on the unconditional partial density plots displayed. The density as of equation (12) is defined heuristically for a small interval $d\eta_T$ as explained in Section 3. Equity values at option maturity and the respective return values are functions of the state variable η_T, the densities with respect to the stochastic Variable η have to be translated into densities of the new variable.

If the distribution function of η for the ABM-case is denoted by $F(\eta_T) = P^{\Omega}(\eta \leq \eta_T; M_T > \eta_B)$ with a density of $f(\eta_T) = \partial/\partial\eta(\Phi(\cdot))$ and equity values at maturity T are an invertible function of EBIT $E_T = E(\eta_T)$, the probabilities for the two events must therefore be the same.

$$P^{\Omega}(\eta \leq \eta_T; M_T > \eta_B) \quad = \quad P^{\Omega}(E \leq E_T; M_T > \eta_B)$$

For the density of equity values f^E this requires that

$$\begin{aligned} f^E(E_T) &= \frac{\partial}{\partial \eta} F(\eta_T) \frac{\partial}{\partial \eta} E^{-1}(\eta_T) \\ &= f(\eta_T) \left(\left. \frac{\partial E_T}{\partial \eta} \right|_{\eta=\eta_T} \right)^{-1}, \end{aligned} \tag{15}$$

since the invertible function is one dimensional in the stochastic parameter.

Transforming the equity density into a return density requires the same transformation as above. Denote the equity return with respect to the expected value at the option maturity by $r_E(T) = \ln(E_T / E^{\mathcal{Q}}(E_T))$. So equity return density can be calculated by

$$f^r(r_E(T)) = f^E(E_T) E_T, \tag{16}$$

since $\partial r_E(T) / \partial E_T = E_T$.

The GBM-case is equivalent to the ABM-case if η is replaced by $\ln(\bar{\eta})$ and η_B by $\ln(\bar{\eta}_B)$. The derivatives in equation 15 are then taken with respect to $\ln(\eta_T)$. Equation 16 does not change.

B Distributional Assumptions and Option Prices

To support the understanding of how distributional assumptions affect option prices, it is necessary to decompose price differences of option with different strikes. Writing the call option value as an expectation under the risk-neutral measure $\mathcal{Q}$

$$\begin{aligned} C(t, T, X) &= E^{\mathcal{Q}}[e^{-r(T-t)}(E_T - X)^+] \\ &= e^{-r(T-t)} \int_0^\infty (E_T - X)^+ q(E_T) dE_T \\ &= e^{-r(T-t)} \int_X^\infty (E_T - X) q(E_T) dE_T, \end{aligned}$$

where E_T denotes the underlying value, X the strike, and T the time of maturity of the call option C at time t, the price difference of a similar call option but with a lower strike $X - \Delta X$, where $\Delta X > 0$ is

$$\begin{aligned}\Delta C(t,T,X,\Delta X) &= e^{-r(T-t)}\bigg(\int_X^\infty (E_T - X)q(E_T)dE_T \\ &\quad - \int_{X-\Delta X}^\infty (E_T - X - \Delta X)q(E_T)dE_T\bigg) \\ &= e^{-r(T-t)}\bigg(\Delta X\int_{X-\Delta X}^\infty q(E_T)dE_T \\ &\quad + \int_{X-\Delta X}^X (E_T - X)q(E_T)dE_T\bigg). \qquad (17)\end{aligned}$$

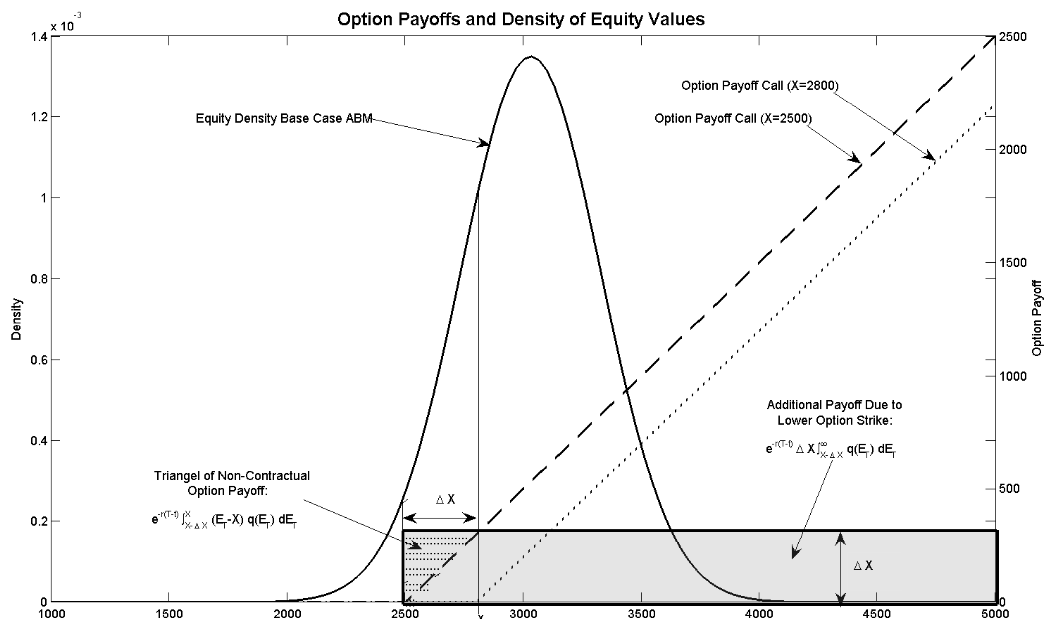

Figure 12: Option price differences due to differences of strike prices

Equation (17) can nicely be illustrated by Figure 12, which shows the payoffs at maturity of two options with strikes X and $X - \Delta X$, respectively. The lower strike option can be replicated as a portfolio of the option with strike X (the area above the gray rectangle and below the dashed line, the area due to the lower strike ΔX (the gray rectangle), and the dotted triangle which the low-strike option holder will not get.

Comparing implicit volatilities is equivalent to comparing the probability mass between $X - \Delta X$ and X. If this probability mass is higher than that of the log-normal distribution, the option price rises more than the Black/Scholes option price leading to an increase in implied volatilities. More generally, the option price difference must exceed

$$\begin{aligned}\Delta C_N(t,T,X,\Delta X) &= e^{-r(T-t)}\left[X\left(N(d_2)-N(d_2-\frac{\gamma}{\sigma\sqrt{T-t}}\right)\right.\\ &\quad -E^{\Omega}(E_T)\left(N(d_1)-N(d_1-\frac{\gamma}{\sigma\sqrt{T-t}}\right)\\ &\quad \left.+\Delta X N(d_2-\frac{\gamma}{\sigma\sqrt{T-t}})\right], \qquad (18)\end{aligned}$$

where $\gamma = 1-(\Delta X)/X$ is the proportional decrease of the strike and d_1 and d_2 are defined as in equation (13). In equation (18), the first two lines represent the dotted triangle of figure 12 and the third line is equivalent to the gray rectangle.

C Additional Figures

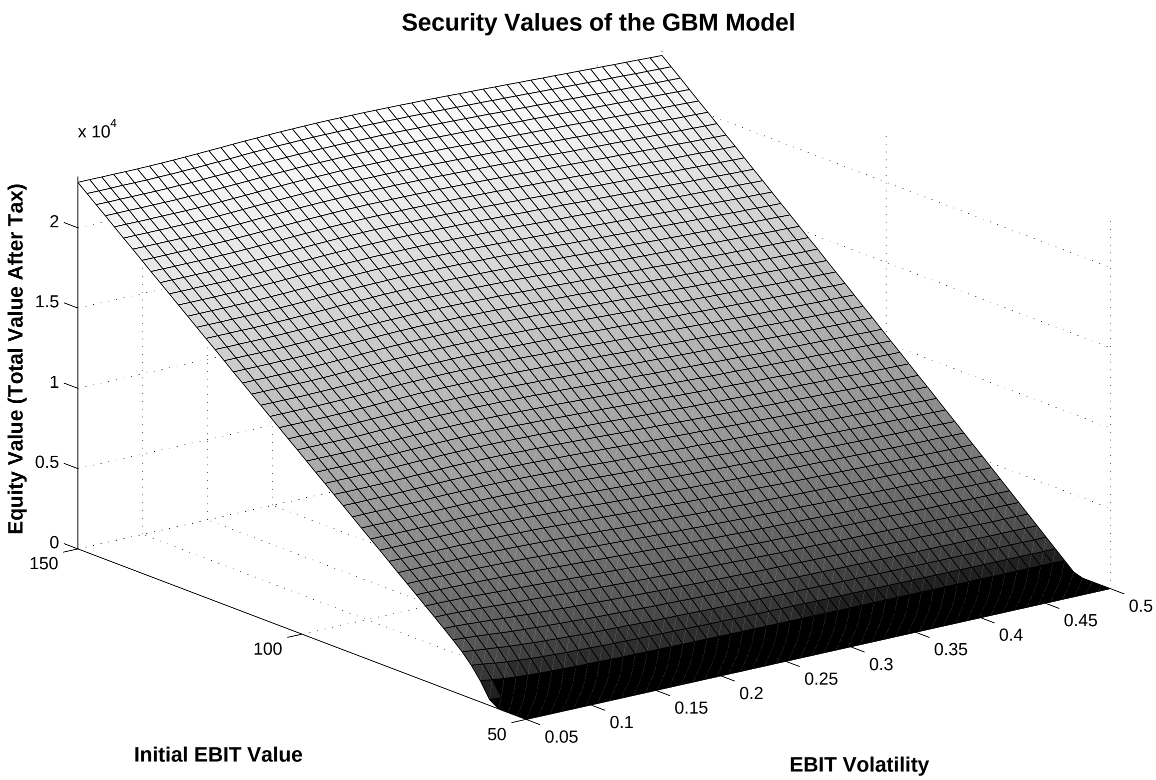

Figure 13: Equity values after tax as a function of EBIT-volatility and current EBIT value of a GBM-firm.

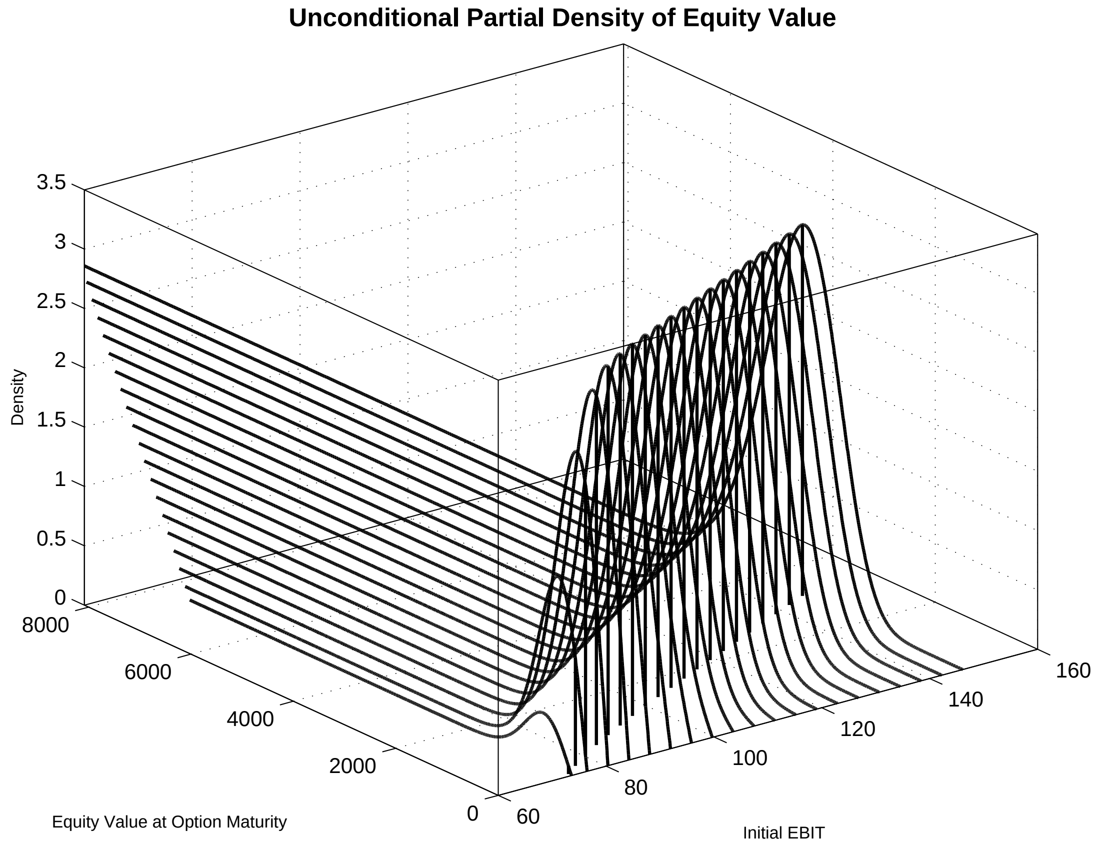

Figure 14: Equity value densities of 6 month equity options in the GBM-Corporate Securities Framework as a function of η_0. Expected equity values are indicated by solid lines. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

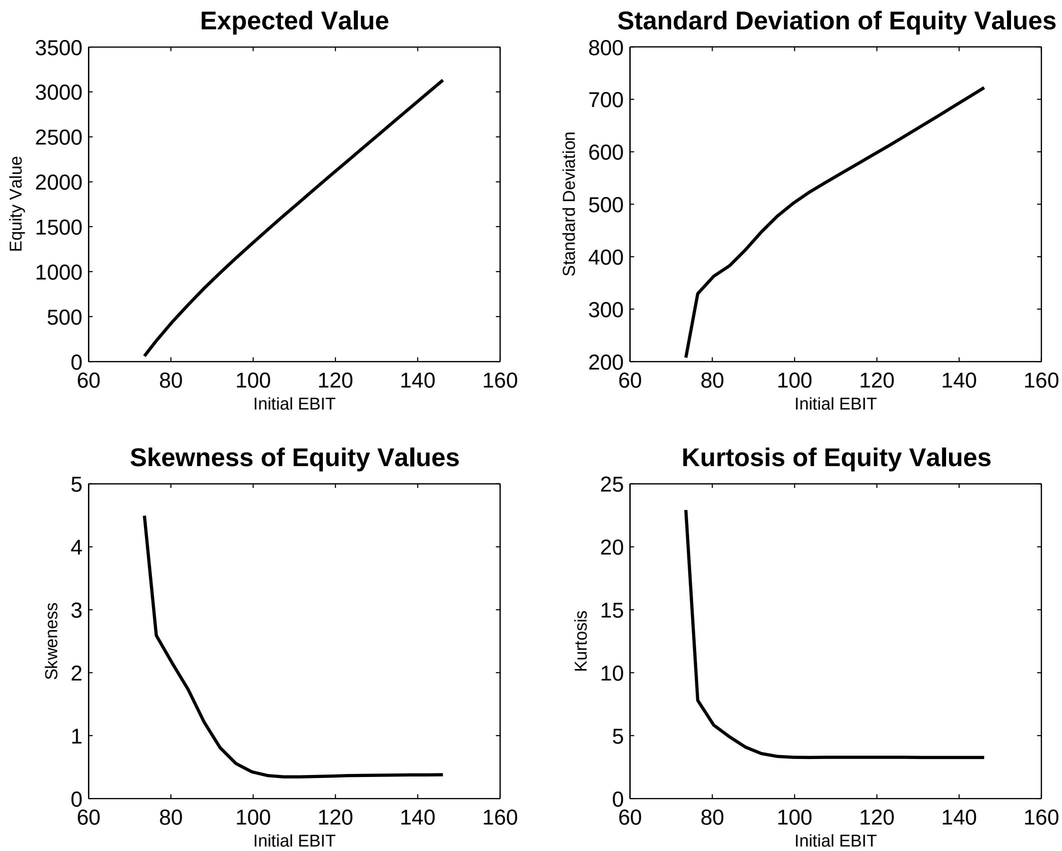

Figure 15: Equity value density moments of 6 month equity options in the GBM-Corporate Securities Framework as a function of η_0. The moments are obtained by numerical integration.

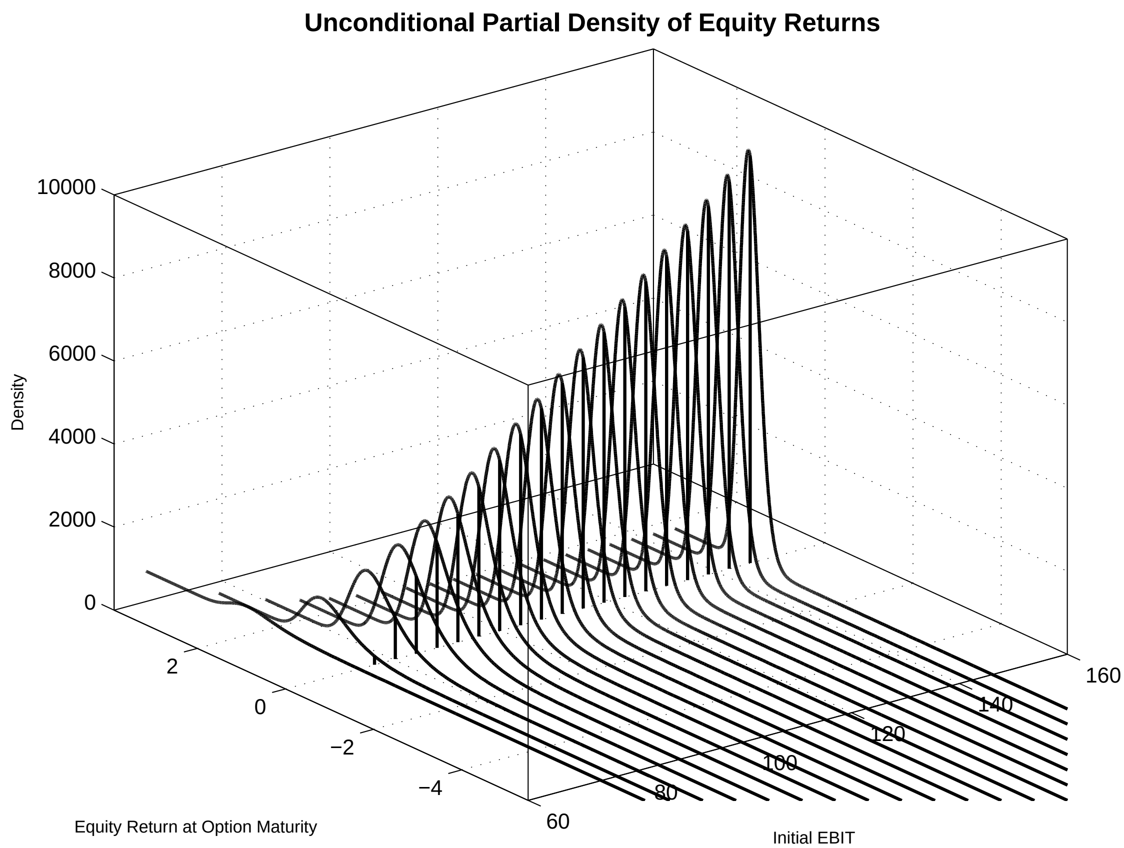

Figure 16: Equity return densities of 6 month equity options in the GBM-Corporate Securities Framework as a function of η_0. The 0-returns are indicated by solid lines. Path probabilities are obtained by differentiating the splined distribution function of EBIT at option maturity.

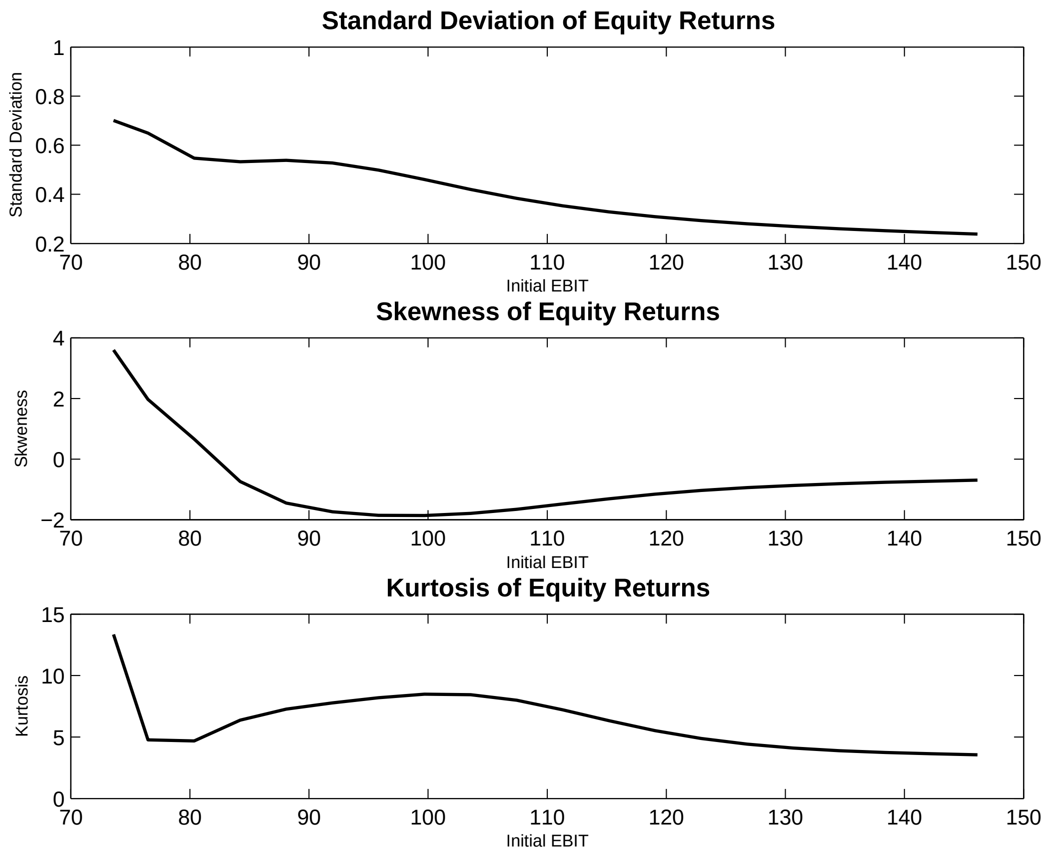

Figure 17: Equity return density moments of 6 month equity options in the GBM-Corporate Securities Framework as a function of η_0. The moments are obtained by numerical integration.