Modelling the Economic Value of Credit Rating Systems

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Abstract

In this paper we develop a model of the economic value of a credit rating system. Increasing international competition and changes in the regulatory framework driven by the Basel Committee on Banking Supervision (Basel II) called forth incentives for banks to improve their credit rating systems. An improvement of the statistical power of a rating system decreases the potential effects of adverse selection, and, combined with meeting several qualitative standards, decreases the amount of regulatory capital requirements. As a consequence, many banks have to make investment decisions where they have to consider the costs and the potential benefits of improving their rating systems. In our model the quality of a rating system depends on several parameters such as the accuracy of forecasting individual default probabilities and the rating class structure. We measure effects of adverse selection in a competitive one-period framework by parametrizing customer elasticity. Capital requirements are obtained by applying the current framework released by the Basel Committee on Banking Supervision. Results of a numerical analysis indicate that improving a rating system with low accuracy to medium accuracy can increase the annual rate of return on a portfolio by 30 to 40 bp. This effect is even stronger for banks operating in markets with high customer elasticity and high loss rates. Compared to the estimated implementation costs banks could have a strong incentive to invest in their rating systems. The potential of reduced capital requirements on the portfolio return is rather weak compared to the effect of adverse selection.

Key words: Rating system, cohort method, Basel, banking regulation, capital requirements, probability of default, adverse selection.

JEL classification: G28, C13

1 Introduction

Increasing international competition and changes in the regulatory framework driven by the Basel Committee on Banking Supervision (Basel II) called forth incentives for banks to improve their credit rating systems. In a competitive framework a poor statistical power of a bank's internal rating system will deteriorate the economic performance due to adverse selection, i.e. customers with a better credit quality than assessed by the bank will potentially walk away and leave the bank with a portfolio of customers with a credit quality lower than estimated. Obviously, improving the statistical power of a rating system will have a positive impact on economic performance. The size of this effect depends mainly on the degree of competitivity of the market environment. The counterweight of these potential benefits are the costs of investing into the power of a rating system such as organizational costs, costs of information technology, and costs of collecting and managing the required data. In addition, a bank's internal rating system with sufficient statistical power might be used for calculating the regulatory capital requirements set by the Basel II Internal Ratings Based Approaches which are expected to be lower than in the Modified Standardized Approach. In addition it can be shown that due to the concave relation between regulatory capital requirements and default probabilities even for banks having already qualified for the Internal Ratings Based Approach a more accurate rating system which enables a finer grained rating class structure leads to lower capital requirements.

It is the main objective of this paper to model the decision whether to invest into the quality of a rating system in a rather general framework. Our model is aimed to quantify the benefits of such an investment. The first part of our analysis is focused on the economic value of increasing the statistical power of a bank's internal rating system. In line with the work by Jordão and Stein (2003) we compare the profitability of prototypical banks with different statistical power of their rating systems in different market environments. In our model the statistical power of a rating system depends on several parameters such as its accuracy and the rating class structure. We measure the accuracy of forecasting individual default probabilities as the variance of the deviations of the forecasted from the actual default probabilities. In this setup this measure is more closely related to the economic impact than the area-under-the-curve measures traditionally used by other researchers.

Many banks use cohort based methods to estimate default probabilities rather than individual estimates based on regression models. Since customers of different credit quality are grouped into cohorts and regarded as being of homogeneous credit quality, additional noise may enter the lending decisions. Thus, the numbers of cohorts used by a rating system and the methods to construct their relative sizes (or, put equivalently, the 'boundaries' between the cohorts) become additional important parameters which describe the statistical quality of a rating system (for the qualitative standards of state-of-the-art rating systems see, e.g., Krahnen and Weber (2001) and Treacy and Carey (2000)).

When examing the profitability of different prototypical banks we assume that banks adopt a full price-based lending approach rather that a cutoff-based approach. In agreement with the findings of Jordão and Stein (2003) we do not expect any influence on the main results of our analysis by this assumption. The cornerstone of our model is the assumption that a bank possesses estimates (not necessarily free of error) of the true individual default probabilities of all its customers. These estimates may be taken from regression based models which yield individual estimates of default probabilities or from cohort methods where the individual estimated default probabilities are set equal to the average default probability of the cohort. A bank prices the loans offered to its customers according to this estimated default probability. More specifically, the spread over the risk-free rate has to cover the expected loss and the proportional ‘general’ costs including operating costs and risk premia related to unexpected losses. For simplicity we assume that the ability to measure unexpected losses is not influenced by the statistical power of the rating system. Note that unexpected losses are likely to be very low for large, well diversified portfolios.

We model the competitivity of the market environment by parametrizing customer elasticity. Customers are assumed to have some better information about their true credit quality. In a full competitive framework with no transaction costs customers who are offered a too high credit spread will eventually walk away to a bank with a more powerful rating system. As a consequence, the bank is left with the customers who are offered a too low credit spread and know about their worse credit quality. This adverse selection effect deteriorates the economic performance of the bank and may lead to insolvency of the bank in extreme cases. However, the fraction of overpriced customers leaving the bank might not be 100% for several reasons. Firstly, there might be imperfect competition among banks due to oligopolistic structures. Secondly, other banks and/or the customers themselves do not have better risk estimates in all the cases. Finally, there are transaction costs for customers willing to leave the bank which might be prohibitively high. To account for all these possible effects we assume that there is a probability that a customer with a better credit quality than assessed does not leave the bank. If this probability is zero we have perfect customer elasticity, if this probability is one there is no competitivity at all.

The second part of our analysis is focused on the impact of the statistical quality of a rating system on regulatory capital requirements which are obtained by applying the current proposal released by the Basel Committee on Banking Supervision. Since internal ratings have to fulfill minimum standards regarding their statistical power we consider only cases with a given accuracy of estimating the default probabilities. Based on the concavity of the

Basel capital requirement function we deduce that the size of the capital charge decreases with the number of rating classes and, given the number of rating classes, the method to construct the rating cohorts may be of particular importance. Again our framework might be useful determining the economic impact of the rating class structure.

Our model provides a framework to quantify potential positive effects of an improvement of the rating system. Of course in real-world decisions the costs of investing into the power of a rating system have to be taken into account. Considering the fact that due to the advent of the new Basel II regulatory framework most rating systems are in their completion stages it is assumed that banks can make realistic estimates of their implementation costs. Moreover, these costs can be considered as independent of the decisions of other institutions and thus are readily quantifiable. Earlier estimations for the German market indicated that for the very small banks featuring total assets of up to ten billion Euro the implementation costs would be around one million Euro (one basis point of total assets), while for the middle sized cooperative institutions these would be in the range of five to seven million Euro (Gross et. al., 2002). Subsequently, estimates have been increased to about five basis points of an institution's total assets (Accenture et al. (2004)), whereas current information suggests that even this high figure will most probably be surpassed. We do not extend our analysis, however, to a formal inclusion of implementation costs. If these costs are known to a bank the decision making process will be obvious. If there is serious uncertainty about the costs then the model will depend heavily on the structure of this uncertainty which is beyond the scope of this paper.

Our analysis is restricted to a partial equilibrium framework. As pointed out by Broecker (1990) effects of adverse selection may lead to a situation where only one bank or one rating system exists in a static general equilibrium framework. Thus, it is a possible extension of our framework to include more than one representative lender and to model the strategic competition between the lenders with different rating systems and different implementation costs. Moreover, in a dynamic setting the timing of the investment decision – the advantage of being first – is an additional interesting question (for a detailed discussion of preemption in a general dynamic game regarding the adoption of new technologies see Fudenberg and Tirole (1985)). The framework presented in this paper describes the basic decision model that might be used in a more sophisticated dynamic setup where modelling the cost differentials of implementing and maintaining rating systems among banks will play a crucial role.

In section 2 we describe the setup of our model. The key ingrediencies are the distribution of individual default probabilities which captures the portfolio structure, the degree of competitivity in the market environment, and the way the accuracy of a rating system is measured. The design of the numerical analysis conducted in this paper is provided in section 3. We described the specific parametrization and the simulation approach used to determine the portfolio returns. Section 4 summarizes the numerical results with respect to the adverse selection effect. Section 5 briefly describes the regulatory framework set by the Basel Committee of Banking Supervision and presents the potential effects of reduced capital requirements. Section 6 concludes the paper.

2 Model Setup

In this section we describe the setup for evaluating the economic impact of rating systems with different predictive power. Many banks are expected to base their PD estimation on the observation of empirical default rates within rating classes. This so-called "cohort method" (see e.g. Jarrow et al. (1997) and Lando and Skodeberg (2002)) is the basic object of our analysis. The main alternative, however, the usage of regression-based forecasts of individual PDs can be seen as a special case of our framework where we have one customer per rating class or - put more precisely - one rating class for each different PD because it is possible to observe customers with identical PDs. In this section we will describe a model to quantify the effect of adverse selection which can be used to indicate if there is an incentive for banks to invest in the improvement of their credit rating systems.

In our setup the credit portfolio of a bank is characterized by the number of customers and the actual or "true" probability of default of each customer. We assume that the recovery rates are known for all customers. So we concentrate on the quality of rating systems with respect to the estimation of PDs. Of course, improving the predictive power of the estimation of recovery rates will also have a significant economic value but we leave this topic open for future research. To simplify the analysis we assume that all exposures are of equal size, which is a reasonable approximation for a large, well diversified portfolio, and that the PDs in the portfolio may be described by a certain ex-ante distribution, which describes the PD distribution of all potential customers for the bank. In our numerical approach the true PD for each customer in the portfolio is drawn from this distribution.

The rating system of the bank will only provide estimates for the true PD of each customer. The difference between the estimated and the true PD will depend on the number and sizes of the rating classes, and the measurement error of the PD. The rating class structure can be freely chosen by the bank. For the purpose of the implementation of our model we will assume that the bank observes a PD of each customer, e.g. by using a logistic regression model, which is not necessarily equal to the true PD and uses this observed PD to slot the customer into a particular rating class. For this slotting the bank has to choose the PD-boundaries to distribute the customers among the rating classes. There is no natural optimal solution to this problem. Different banks will divide up their customers according to different heuristic rules, e.g. approximately equal number of customers per rating cohort. In the case of infinitely many rating classes this slotting is not necessary. This method is then equivalent with using the observed PDs directly from the regression model.

Once the customers are slotted into rating classes the bank estimates the PD of each rating class and uses this PD for pricing and risk management of the customers of this rating class. The estimated PD of each rating class is taken as the expected number of defaults divided by the number of customers. Under the assumption of a stationary PD distribution in the portfolio this expected default rate should on average equal the actual or observed default rate of the past period which is the usual basis for PD calculation. Thus this approach resembles the procedures applied by many banks as well as by rating agencies like Standard&Poor's or Moody's to provide PDs based on historic default data related to their rating classes.

If we assume no measurement errors the choice of only one rating class will always maximise the difference over all individual customers between true and estimated PD. Provided that individual true PDs differ across customers the deviations of individual true PDs from the overall average PD of the portfolio (which equals the estimated PD of the single rating class) are obviously higher than deviations from subgroup averages. Thus without measurement errors the bank can reduce the difference between estimated and true PD by using more and dispersed rating classes.

In the next step we introduce measurement errors for the observed PDs of each customer which are used for slotting into the rating classes. Since PDs are defined between zero and one and usually close to zero the use of an additive normally distributed error term has the disadvantage of significant probabilities of negative PDs. To avoid this problem we transform the PDs into credit scores according to the relation in a logistic regression framework (see equation 1) and we assume that these credit scores rather than the PDs are shifted by error terms that are normally distributed with mean zero (see equation 2). Note, that we introduce a credit score as a suitable nonlinear transformation of the PD for technical reasons only. This approach allows the use of normally distributed error terms in a convenient and economically reasonable way. However, any other suitable assumption about the error term (e.g., muliplicative error terms) could have been used.

$$PD_{true} = \frac{1}{1+e^{-credit\ score_{true}}} \Leftrightarrow credit\ score_{true} = ln\left(\frac{1 - PD_{true}}{PD_{true}}\right) \qquad (1)$$

$$PD_{observed} = \frac{1}{1+e^{-(credit\ score_{true}+\varepsilon)}} \qquad \text{with} \qquad \varepsilon \sim N(0,\sigma^2) \qquad (2)$$

The parameter σ controls the magnitude of the estimation error. Introducing measurement errors means that there will be differences between the true PD and the observed PD for some customers and therefore these customers are potentially slotted into the wrong rating class. Since the estimated PD of each rating class is taken as the expected number of defaults divided by the number of customers higher measurement errors will increase the probability that customers with high PDs are slotted in low risk rating classes and vice versa. Thus the estimated PD for low risk rating classes will be higher and the estimated PD for high risk rating classes will be lower compared to rating systems without measurement errors. Therefore banks with measurement errors will have less accurate estimated PDs and will be stronger exposed to adverse selection.

The number and sizes of rating classes and the parameter σ of the measurement error are under the control of the bank. Investing into the predictive power of a rating system thus means to be able to reduce the measurement error and to use more und better dispersed rating classes. The first goal of this paper is to analyse the effect of these parameters on the return of different portfolios and in different market situations to obtain the optimal strategy concerning the investment into the rating system.

Since we want to apply a risk-adjusted pricing we first define the loan pricing mechanism. We assume that the bank needs to receive an interest rate *r* to cover all 'general' costs besides credit risk related to the expected loss. The general costs include operating costs and risk premia related to unexpected losses. We assume that the ability to measure unexpected losses is not influenced by the decisions in our model. Credit risk related to expected losses is priced by demanding a credit spread *s*. For simplicity we restrict our analysis to one-period bonds. The credit spread depends on the estimated PD and the loss-given-default (LGD) of the individual exposures. We assume that the LGD is estimated according to the Basel II definition meaning that *(1-LGD)* is the recovery for the whole loan, i.e. principal and interest including the spread. If no default occurs the bank receives *(1+r+s)*, if default occurs the bank receives *(1+r+s)·(1-LGD)*. The latter payoff is justified by two assumptions: (i) The loss rate is measured with respect to principal plus interest, and (ii) default occurs at the end of the period. These assumptions are also in line with the 'fractional recovery of market value' assumption inherent in many reduced form credit risk models (see, eg, Duffie and Singleton (1999)) However, the impact of this assumption on the pricing relationship explained below is negligible. In assuming the LGD to be constant in line with the assumption of the foundation IRB approach the credit spread is a function of PD, LGD, and *r*. In essence, the expected payoff of the loan has to be equal to the riskfree payoff of *1+r*:

$$1+r=(1-PD)\cdot(1+r+s)+PD\cdot(1-LGD)\cdot(1+r+s) \qquad (3)$$

Solving for the credit spread s yields:

$$s = (1 + r) \cdot \frac{PD \cdot LGD}{1 - PD \cdot LGD} \tag{4}$$

Using this pricing mechanism we are now able to introduce our concept of adverse selection. The rating system of a bank provides the estimated PD for each customer who applies for a loan. Using this PD and the LGD the credit spread which is offered to customers can be calculated by the pricing mechanism. If the PD is overestimated the customer will be offered a credit spread, which is too high compared to her true PD. We will assume that customers, who are offered a too high spread, will leave the bank with a probability which is dependent on the magnitude m of the deviation from the spread corresponding to their true PD.

$$m = s_{estimated} - s_{true} = (1 + r) \cdot \frac{PD_{estimated} \cdot LGD}{1 - PD_{estimated} \cdot LGD} - (1 + r) \cdot \frac{PD_{true} \cdot LGD}{1 - PD_{true} \cdot LGD} \tag{5}$$

There are several possible reasons why a customer might not leave the bank in a situation where she is offered a too high spread. First of all, one can imagine that a customer is not better informed about her default probability than the bank. A second reason could be related to the degree of competitivity and segmentation in the market. It is possible that other banks do not have better rating systems as well and do not offer more attractive spreads to an informed customer who is willing to leave her bank. Finally, transaction costs and cross selling effects may also serve as an additional reason for customers to pay higher spreads to their bank. Since we do not want to separate among these effects it suffices to model the outcome of the customers' decisions using a simple probability distribution. The probability to leave the bank is dependent on m and on the elasticity of the customer. To model the elasticity of the customer we assume the following functional relation between the probability of leaving the bank and the estimation error in the spread

$$probability\ to\ leave = 1 - e^{-\alpha \cdot m} \tag{6}$$

where α is a elasticity parameter. If α is zero, all customers will stay. If α goes to infinity, all customers with an overestimated PD will leave the bank. All customers who are offered a spread corresponding to their true PD or a lower spread will stay with the bank.

This probability to leave the bank models the impact of adverse selection. We will analyse the effects using different degrees of elasticity. If the bank is able to estimate the true probability of default of each customer using its rating system it will on average earn an interest rate of r on the portfolio. If the estimated and true PD differs the return on the portfolio will be lower than r dependent on the elasticity of the customers, which generates the effect of adverse selection.

Given the customer elasticity the bank observes which customers form the actual portfolio and can now evaluate the return of its portfolio $r_{portfolio}$ which is the average over the returns on the individual loans r_i. The return r_i is dependent on whether the customer i defaults or not, which will happen with the individual true PD of the customer:

$$1 + r_i = \begin{cases} 1 + r + s & no\ default\ of\ customer\ i \\ (1 + r + s) \cdot (1 - LGD) & default\ of\ customer\ i \end{cases} \tag{7}$$

$$r_{portfolio} = \frac{1}{n} \sum_{i=1}^{n} r_i \tag{8}$$

The return of the portfolio will depend on the specific parameter values. In the following section we present potential parametrizations for this model setup and in section 4 we numerically analyse the influence of certain parameters, e.g. customer elasticity, on the portfolio return.

3 Design of numerical analysis

In this section we briefly explain the simulation approach based on the model setup described in the previous chapter. First of all we want to quantify the effect of adverse selection for different portfolios. A portfolio is described by the number of customers and their individual "true" probability of default. In the numerical analysis we fix the number of customers at 10,000. This number represents a well diversified portfolio implying that the observed loss is likely to be close to the expected loss. After fixing the number of customers we have to determine their true PDs. As explained in section 2 the true PD for each customer in the portfolio is drawn from a certain ex-ante distribution, which describes the PD distribution of all potential customers for the bank. We choose to use the Beta distribution for this analysis because it is easy to handle and has some attractive properties, e.g. it is defined over a finite interval as PDs are and it allows for extreme skewness as we expect for PD distributions (see Renault and Scaillet (2003) on the use of Beta distributions to model recovery rates). We also calibrate different distributions, e.g. Gamma distributions and Log-normal distributions, on our set of default data (see below). The Beta distribution yields the highest likelihood using maximum likelihood estimation. However the results for other distributions are quite similar. Thus our results should not depend on the specific choice of Beta distributions. To compare the effects of adverse selection for portfolios of different quality three Beta distributions have been chosen, from which the PDs are drawn:

Good portfolio: Beta distribution with $p = 0.4$ and $q = 19$ (median PD = 0.77%)

> These parameters are chosen such that the distribution represents a quite good corporate portfolio. Compared to the average portfolio this means that we observe more customers with very small PDs.

Average portfolio: Beta distribution with $p = 0.7$ and $q = 37.6$ (median PD = 1.08%)

> These parameters are estimated by using a dataset of more than 30,000 Austrian corporate customers provided by Creditreform (see Schwaiger (2003)). Employing a logistic regression we estimate individual PDs for each customer in the dataset. We use a maximum likelihood estimation to find the parameters for the Beta distribution which optimally explains the individual PDs. Thus, this distribution represents a real portfolio.

Weak portfolio: Beta distribution with $p = 1.4$ and $q = 58$ (median PD = 1.84%)

> These parameters are chosen such that the distribution represents a rather weak corporate portfolio. Compared to the distribution of the average portfolio this means that we observe more customers with high PDs.

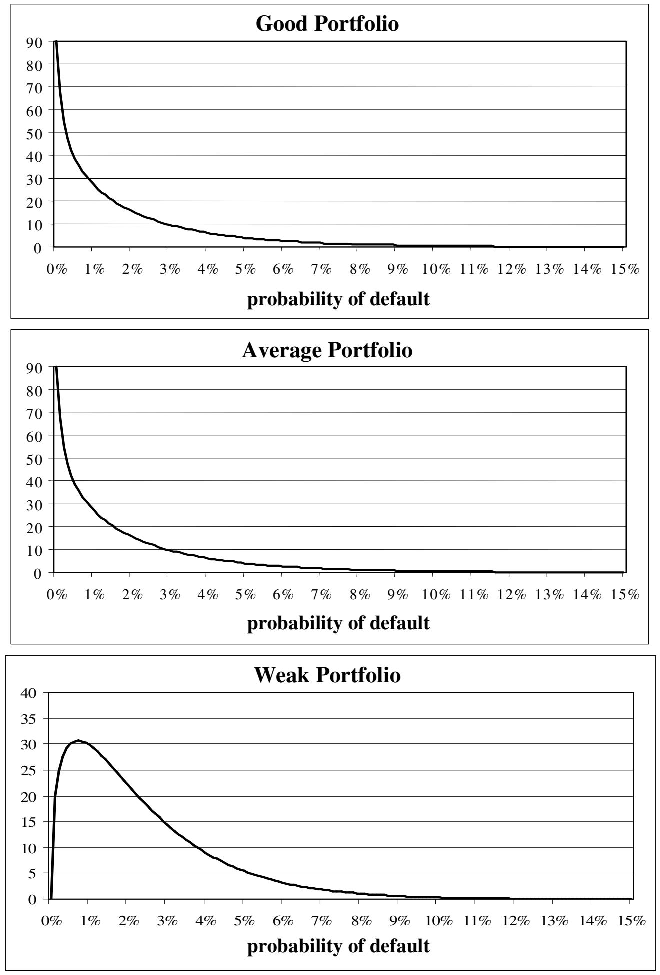

Figure 1: Densities of the Beta distributions representing a good (p = 0.4 and q = 19), an average (p = 0.7 and q = 37.6), and a weak (p = 1.4 and q = 58) portfolio. The parameters of the average portfolio are calibrated to a dataset of more than 30,000 Austrian corporate customers provided by Creditreform (see Schwaiger (2003)).

In this context the notion of a 'weak' and a 'good' portfolio has a relative meaning. The portfolios are weak and good related to the empirically observed PD of our dataset which consists of a large sample of Austrian corporate customers. However, there are real-world portfolios which are better (e.g., sovereign lending) or weaker (e.g., credit cards) in absolute terms.

Given the number of customers and their true PDs we generate the PDs the bank observes for slotting the customers into rating classes. This is achieved by transforming the true PD drawn from the relevant Beta distribution to a true credit score using equation (1) and by adding a simulated measurement error for each customer as described in equation (2). The magnitude of the measurement error of the rating system is controlled by the parameter σ (see equation 2). In this paper we use four levels for the magnitude of the estimation error which represent different levels in the development process of a Basel II compliant rating system. These four levels should represent a wide variety of degrees of accuracy that can be observed for rating systems in the banking industry. The numbers for σ that we use are calibrated to data of the Austrian Major Loans Register provided by the Austrian National Bank where all banks have to report the sizes of major loans along with the rating of the customers and the documentation of the rating system. Thus this data set contains the ratings of different banks for identical customers. From this cross-sectional information we infer the dispersion of ratings and PDs for individual customers and calibrate the magnitude of the measurement error σ. According to the results of this analysis we could identify three different groups of banks in the data set where we summarize the qualitative information about the rating systems and the respective values for the measurement error σ:

- Low accuracy ($\sigma = 2$): The bank has recently started to develop its rating system for estimating PDs. The rating system is not calibrated to default data and is only determined by qualitative judgement.
- Medium accuracy ($\sigma = 0.5$): The bank has one or two years of experience and the rating system is calibrated to this short history of default data.
- High accuracy ($\sigma = 0.1$): The bank has three to fours years of experience and the rating system has been improved through rating validation.

Additionally to these three groups we use a fourth group because even the most developed rating systems in our data set can be further improved over time:

- Perfect accuracy ($\sigma = 0$): The bank has the experience of at least one full economic cycle and the rating system is improved through repeated rating validation (no bank fulfilled these criteria in our data set).

In the next step the bank has to choose the number and sizes of the rating classes. We will consider banks which use one, two, five, ten, and infinitely many rating classes. These categories should basically cover all numbers of rating classes used in the banking sector and are linked to the information provided by rating agencies (e.g. Standard&Poor's or Moody's) in the following way:

- One rating class: The bank only uses the average default rate of the whole portfolio.
- Two rating classes: The bank distinguishes between investment grade (AAA to BBB) and speculative (BB to CCC/C) grade customers.
- Five rating classes: The bank uses the number of rating classes of the Basel II standardized approach (rating 1: AAA and AA, rating 2: A, rating 3: BBB, rating 4: BB and B, rating 5: CCC/C)
- Ten rating classes: This is comparable to a bank which uses all main S&P rating classes (AAA to C) without modifiers.
- Infinitely many rating classes: The bank directly uses the observed PD of each customer. For the empirical results this is comparable to the use of all twenty S&P rating classes (using all modifiers).

Concerning the sizes of the rating classes there is no natural best solution to define PD boundaries. For the numerical analysis we propose four different methods, which seem to be reasonable ways in defining the sizes of rating classes (see appendix).

After slotting the customers in rating classes by using the observed PDs the bank estimates the PD of each class. This estimated PD is taken as the expected number of defaults divided by the number of customers (see section 2) and is used for pricing the loan of each customer in this rating class.

The next parameter, which is necessary for the bank to price loans, is the LGD. We will assume that the LGD is equal for all customers and known to the bank. The effect of the LGD estimation is an interesting topic, but in this paper we want to focus on the effects of the PD estimation. Nevertheless we will observe the effects of adverse selection for three different LGD-levels in line with the Basel II framework:

- High (75%): The Basel II IRB foundation approach sets the LGD to 75% for subordinated unsecured loans.
- Medium (45%): The Basel II IRB foundation approach sets the LGD to 45% for senior unsecured loans.
- Low (25%): This is consistent with typical senior loans which are completely secured by real estate (→ in the Basel II IRB foundation approach complete securitisation by real estate reduces the LGD to 35%) and additional provide some eligible financial collateral.

Having an estimate for PD and LGD the bank can calculate the credit spread for each customer by equation (4) given some interest rate *r*, which covers all costs besides credit risk related to the expected loss. We set this interest rate *r* to 3%, but the results are virtually the same for any other reasonable level of *r*.

The customers decide then whether to accept or to reject the loan offered to them. Every customer, who is offered the spread corresponding to her true PD or a lower credit spread, will accept the loan. All customers offered a higher credit spread will reject the loan with a probability depending on the elasticity parameter α and on the magnitude *m* of the deviation. In this paper we define three levels for the customer elasticity (low: α = 100, medium: α = 500, high: α = 10,000). We set the values for α in a way that it potentially covers most real world scenarios. Since to our knowledge there is no research published about the empirical relationship between credit spread and customer behavior and we do not have access to empirical data to estimate α, we have to restrict ourselves to choices of 'plausible' values of α which might be judged by the probabilities of a customer to leave a bank implied by certain levels of α given in table 2 below. However, provided sufficient empirical data on credit spreads and customer behavior α might be calibrated to these data. The exit and voice framework introduced by Hirschman (1970) seems to be an adequate starting point for solving this problem.

	elasticity		
deviation of spread	**low**	**medium**	**high**
+ 5 bp	4.9 %	22.1%	99.3%
+ 10 bp	9.5%	39.3%	99.9%
+ 50 bp	39.3%	91.8%	100.0%

Table 2: Probability to leave the bank for given levels of elasticity. Probabilities are presented for different deviations of the offered spread from spread corresponding to the true PD (left column) and for low (α = 100), medium (α = 500), and high (α = 10,000) elasticity, respectively.

Given the level of elasticity we simulate which customers leave the portfolio and which stay. Through adverse selection even with a low level of elasticity the bank will loose some customers. After determining which customers form the actual portfolio of the bank, we are now ready to calculate the return of this portfolio $r_{portfolio}$ which is the average over the returns on the individual loans r_i (see equations 7 and 8). To calculate r_i we simulate which customers in the portfolio actually default using their individual true PDs. The return of the portfolio is the result of one simulation path. For each combination of the parameters we run 100 simulations to estimate the average of the portfolio returns. These average returns are the main results of our numerical analysis. We are able to examine the portfolio return effects for different parameter constellations. The main task to compare rating systems with different predictive power can now be achieved by simulating their returns in the proposed way.

4 Numerical results

In this section we quantify the effect of rating systems with different predictive power on the portfolio return. Investing into a better rating system means to use more and better dispersed rating cohorts and to reduce the measurement error in the estimation of the default probabilities. In the first analysis we want to concentrate just on the effect of decreasing the measurement error. To do this we define one base case:

Base Case: number of rating cohorts: 10
sizes of the rating cohorts: linearly increasing number of defaults (method 4)
number of customers: 10,000
LGD: medium (45%)
elasticity: medium (α = 500)

For this base case we quantify the effect on the portfolio return when improving the accuracy of the PD estimation for the three different portfolios of different customer quality (see section 3). In the first step the simulation approach of section 3 is used to estimate the portfolio return for low accuracy (σ = 2), medium accuracy (σ = 0.5), high accuracy (σ = 0.1), and perfect accuracy (σ = 0) given the parameter constellation of the base case. In the second step the increase Δ of the portfolio return when using a rating system with medium, high, or perfect accuracy level instead of a rating system with low accuracy level is calculated:

$$\Delta_{medium} = r_{portfolio,\ medium\ accuracy} - r_{portfolio,\ low\ accuracy} \quad (9)$$

$$\Delta_{high} = r_{portfolio,high\ accuracy} - r_{portfolio,\ low\ accuracy} \quad (10)$$

$$\Delta_{perfect} = r_{portfolio,\ perfect\ accuracy} - r_{portfolio,\ low\ accuracy} \quad (11)$$

The increase Δ represents the potential gain in portfolio return for a bank when improving a rating system with low accuracy to medium, high, or perfect accuracy. Table 3 shows the increase Δ of the portfolio return when improving a rating system for the base case:

base case	**accuracy of PD estimation**		
	medium	**high**	**perfect**
good portfolio	30.8	43.7	44.8
average portfolio	32.6	45.9	46.8
weak portfolio	39.0	56.4	58.7

Table 3: Increase in portfolio return (in bp) for the base case when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

For realistic portfolios the return increases by 30 to 40 bp p.a. when the bank upgrades its rating system from low to medium accuracy. Improving from medium to high accuracy increases the portfolio return still by approximately 15 bp. Moving from high to perfect accuracy results only in an approximate 1 bp improvement.

These results indicate that an improvement of the rating system has a very strong effect for banks with low or medium accuracy systems. Avoiding the effect of adverse selection improves the portfolio return significantly. The effect is also stronger for banks with rather weak portfolios.

In the next step we present results of the portfolio return where the parameter values of our base case are changed. We want to analyse if banks with certain characteristic in their portfolio (e.g. highly collaterised loans) have more incentives to invest in their rating system.

In order to check for the influence of the degree of competitivity in the market environment we change the elasticity parameter of the base case. One would expect that for banks with more elastic customers, i.e. who leave the bank with higher probability if they are offered a too high credit spread, the improvement of the rating system is more important. Table 4 and 5 show the increase Δ of the portfolio return when improving a rating system with low accuracy to medium, high, or perfect accuracy in the case of high and low elasticity:

high elasticity	**accuracy of PD estimation**		
	medium	**high**	**perfect**
good portfolio	32.4	49.8	53.7
average portfolio	34.2	51.6	55.8
weak portfolio	36.1	58.9	62.9

Table 4: Increase in portfolio return (in bp) given high customer elasticity (α=10,000) when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

low elasticity	**accuracy of PD estimation**		
	medium	**high**	**perfect**
good portfolio	18.6	25.3	26.1
average portfolio	19.7	26.4	27.3
weak portfolio	25.2	32.7	33.8

Table 5: Increase in portfolio return (in bp) for a low customer elasticity (α=100) when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

The results clearly show that in loan markets with higher customer elasticity the improvement is significantly stronger. In markets with oligopolistic structures and high market power for a bank the adverse selection effect is not that important but still around 20 bp.

Next we analyse the effect of the LGD on the improvement potential of the rating system. We compare portfolios with high LGD (75%) and low LGD (25%). We would expect that the improvement of the rating system is more important for portfolios with high LGD. Table 6 and 7 show the increase Δ of the portfolio return when improving a rating system with low accuracy to medium, high or perfect accuracy in the case of high and low LGD:

high LGD	**accuracy of PD estimation**		
	medium	**high**	**perfect**
good portfolio	53.9	78.9	84.0
average portfolio	55.0	80.8	84.6
weak portfolio	62.0	96.8	102.3

Table 6: Increase in portfolio return (in bp) for a high LGD (75%) when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

low LGD	**accuracy of PD estimation**		
	medium	**high**	**perfect**
good portfolio	16.3	21.8	22.1
average portfolio	16.9	22.8	22.9
weak portfolio	21.6	29.4	29.6

Table 7: Increase in portfolio return (in bp) for a low LGD (25%) when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

The results indicate that the LGD is very important for the size of the effect of improving rating accuracy. Banks with low LGD, e.g. due to highly collaterised loans, do not depend that much on the quality of their PD estimation. On the other side banks with completely uncollaterised loans depend heavily on the accuracy of their PD estimation.

In the last analysis we compare the effect of rating accuracy for a different number of rating cohorts. In the base case we used ten rating cohorts. Now we use five and infinitely many ratings cohorts to analyse the effect.

five rating cohorts	**accuracy of PD estimation**		
	medium	**high**	**perfect**
good portfolio	28.6	40.5	40.8
average portfolio	29.7	41.4	41.6
weak portfolio	34.9	50.2	50.6

Table 8: Increase in portfolio return (in bp) for five rating cohorts when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

infinitely many rating cohorts	**accuracy of PD estimation**		
	medium	**high**	**perfect**
Good portfolio	32.2	46.8	47.7
average portfolio	34.3	47.9	49.4
Weak portfolio	41.8	60.2	63.3

Table 9: Increase in portfolio return (in bp) for infinitely many rating cohorts when improving the accuracy of the rating system from a low accuracy level (σ=2) to a medium (σ=0.5), high (σ=0.1), or perfect (σ=0) accuracy.

As expected, the effect of rating accuracy is more important for banks that use more rating cohorts. Banks that use a low number of rating cohorts have only a rough measure for the PDs of the customers which are average default rates of coarse cohorts even if they can measure individual PDs without error. Thus improving rating accuracy is not that important for their situation.

5 Capital requirements

In this section we analyse the effect of improving the rating system on the regulatory capital requirements of a bank. The Basel Committee on Banking Supervision previously has released a series of consultative documents, accompanying working papers, and finally the new capital adequacy framework commonly known as Basel II. One of the cornerstones of this new Capital Framework ("Pillar 1") is a new risk-sensitive regulatory framework for a bank's own calculation of regulatory capital for its credit portfolio. Banks which qualify themselves in terms of data availability, statistical methods, risk management capabilities, and a number of additional qualitative requirements will be allowed to adopt the Internal Rating Based (IRB) approach to calculate their capital requirements. In the Foundation IRB (FIRB) approach banks can use their own PD estimates of their customers. In the Advanced IRB approach they can use own estimates of average loss rates and credit conversion factors additionally.

We do not focus on institutional changes in regulatory capital related to differences in the formulas for the risk weighted capital in the Modified Standardized Approach and the FIRB. We concentrate rather on the economic value of improving a rating system given that a bank has already qualified itself for the FIRB approach.

In the FIRB approach banks are allowed to use internal PD estimates to calculate the effect on capital requirements for different rating systems using the proposed formulas suggested by the Basel Committee. For expository purposes we concentrate on the formula for corporate customers but the essence of our results will hold for other customer classes (retail, banks, sovereigns) as well. The proposed capital requirement (CR) of a standard uncollateralized corporate exposure expressed as a function of the customer's PD consists of two parts. The first part represents the capital requirement for the unexpected loss (CR_UL):

$$CR\_UL(PD) = \left[LGD \cdot N\left(\sqrt{\frac{1}{1-R(PD)}} \cdot G(PD) + \sqrt{\frac{R(PD)}{1-R(PD)}} \cdot G(0.999) \right) - PD \cdot LGD \right] \cdot \\ \cdot \frac{1}{1-1.5 \cdot b(PD)} \cdot \left(1 + (M-2.5) \cdot b(PD)\right) \tag{12}$$

with

$$R(PD) = 0.12 \cdot \frac{1-e^{-50 \cdot PD}}{1-e^{-50}} + 0.24 \cdot \left(1 - \frac{1-e^{-50 \cdot PD}}{1-e^{-50}}\right) \tag{13}$$

$$b(PD) = (0.11852 - 0.05478 \cdot log(PD))^2 \tag{14}$$

where *N(.)* Standard normal cumulative distribution function

G(.) Standard normal inverse cumulative distribution function

LGD Loss-given-default; in the FIRB the LGD is set equal to 45% for standard uncollateralized corporate exposures

PD For corporate exposures we have *PD = max(PD*; 0.03%)*, where PD* denotes the estimated PD of the customer

M Effective maturity; in the FIRB the effective maturity is set equal to 2.5 years.

The second part represents the capital requirement for the expected loss (CR_EL):

$$CR\_EL(PD) = PD \cdot LGD - total\ eligible\ provisions \tag{15}$$

For our analysis we set the total eligible provisions to zero. As long as the provisions of customers are equal this assumption does not affect the results at all. The capital requirement for a customer is then the sum of the capital requirement for expected and unexpected loss (see figure 2):

$$CR(PD) = CR\_EL(PD) + CR\_UL(PD) \tag{16}$$

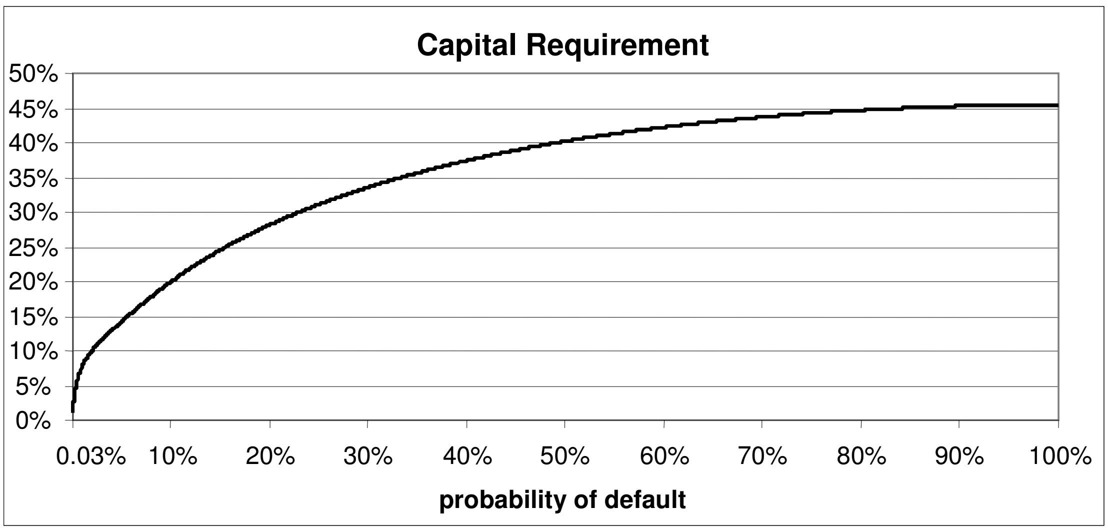

Figure 2: Capital requirement function for standard uncollateralized corporate exposures.

For the capital requirement the PDs of the customers have to be estimated. We will use PDs which are measured without error to take into account the quality standards under the Capital Framework. Thus the accuracy of a rating system is represented by the number and sizes of the rating classes. It is our objective to measure the effect of these two parameters on the capital requirement of certain portfolios. An investment in the rating system means to be able to divide up the portfolio into more and dispersed rating classes according to the quality standards of Basel II.

Consider a rating class defined by an arbitrary PD interval. The capital requirement for this rating class is obtained by

$$CR(E[PD_i,\ i \in \text{rating class}]) \tag{17}$$

where CR(.) denotes the capital requirment function and $E[PD_i,\ i \in \text{rating class}]$ denotes the expected (or average) PD of rating class i. If the rating class i is divided into two non-empty subclasses the capital requirement is given by

$$CR(E[PD_j,\ j \in \text{rating subclass 1}]) + CR(E[PD_k,\ k \in \text{rating subclass 2}]) \tag{18}$$

The function to calculate the capital requirements out of the PDs, which is given in (12-16), is strictly concave for PD values greater than the "floor" PD of 3 bp. As a consequence, it follows from Jensen's inequality that

$$CR(E[PD_i,\ i \in \text{rating class}]) > CR(E[PD_j,\ j \in \text{subclass 1}]) + CR(E[PD_k,\ k \in \text{subclass 2}]) \tag{19}$$

Thus, having an iterative application of this argument in mind we conclude that the finer the rating system the lower the regulatory capital requirement. In figure 3 we show the difference in the capital requirements when we have two customers (*A* and *B*) and the PD is firstly estimated for each customer and then the PD is estimated for the portfolio of the two customers.

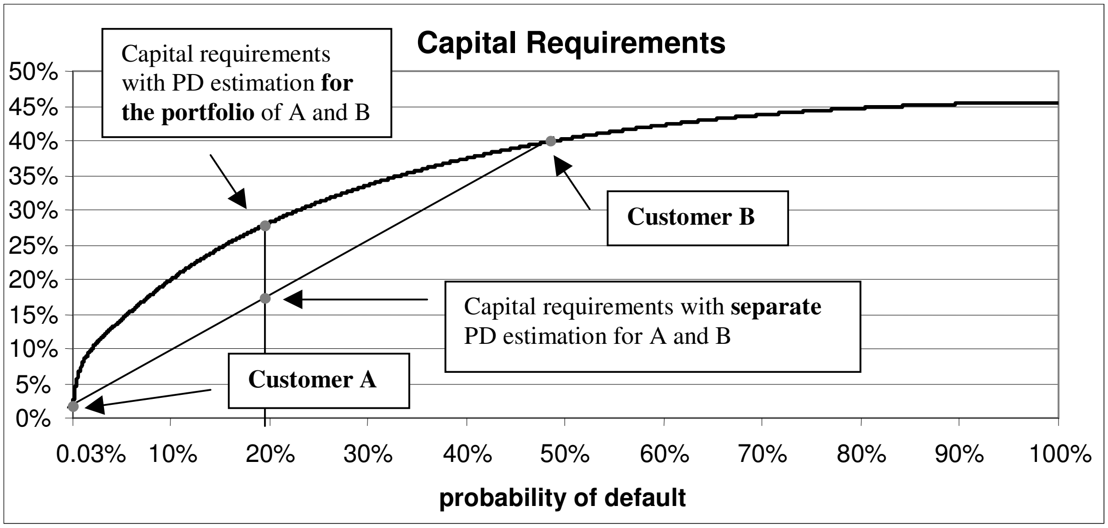

Figure 3: Consequences of Jensen's inequality on the calculation of the capital requirements.

From the theoretical point of view we can deduct that the structure of a rating system has a potential impact on a financial institution's capital requirements. The main result based on Jensen's inequality is that the finer the rating system, the lower the capital requirements. However, we cannot deduct any indication about the potential size of these effects. To provide such quantification we make more specific assumptions about the distribution of PDs using the three different portfolios described in section 3.

Given the Beta density for all potential customers we can infer how many percent of the customers in the portfolio have a PD inside the interval $[x_1, x_2]$ and we can obtain the expected PD for this group of customers analytically. This is all we need to know to calculate the capital requirement for a set of rating cohorts where the corresponding cohort boundaries are given by their minimal and the maximal PDs.

The expected value for the PD of a customer given that the customer has a PD between x_1 and x_2 has the following functional form

$$E[PD \mid x_1 \leq PD \leq x_2] = \frac{p}{p+q} \cdot \frac{cdf(p+1,q,x_2) - cdf(p+1,q,x_1)}{cdf(p,q,x_2) - cdf(p,q,x_1)} \tag{20}$$

where *cdf(.)* denotes the cumulative density function of a Beta distribution with parameters p and q. Provided this analytical relation we can avoid a simulation procedure compared to section 4.

We calculate the capital requirement for the three portfolios under consideration using one, two, five, ten, and infinitely many rating cohorts and using the four different methods to construct credit score intervals (see appendix). Applying these methods of defining the sizes of the rating classes the capital requirement for a different number of rating classes can be compared. Tables 10 to 12 show the resulting capital requirements for the four methods over the different PD distributions of our three portfolios:

good portfolio	**1 cohort**	**2 cohorts**	**5 cohorts**	**10 cohorts**	**∞ cohorts**	**reduced capital requirements using 10 instead of 5 cohorts**
method 1	9.56%	9.55%	9.29%	8.84%	7.30%	45 bp
method 2	9.56%	7.91%	7.47%	7.36%	7.30%	11 bp
method 3	9.56%	8.94%	8.22%	7.87%	7.30%	35 bp
method 4	9.56%	8.63%	7.75%	7.48%	7.30%	27 bp

Table 10: Capital requirements for the good portfolio using one, two, five, ten, and infinitely many rating cohorts for different methods of defining the sizes of the rating cohorts.

average portfolio	1 cohort	2 cohorts	5 cohorts	10 cohorts	∞ cohorts	reduced capital requirements using 10 instead of 5 cohorts
method 1	9.37%	9.37%	9.25%	8.93%	8.00%	32 bp
method 2	9.37%	8.43%	8.11%	8.04%	8.00%	7 bp
method 3	9.37%	8.90%	8.46%	8.27%	8.00%	19 bp
method 4	9.37%	8.69%	8.21%	8.08%	8.00%	13 bp

Table 11: Capital requirements for the average portfolio using one, two, five, ten, and infinitely many rating cohorts for different methods of defining the sizes of the rating cohorts.

weak portfolio	1 cohort	2 cohorts	5 cohorts	10 cohorts	∞ cohorts	reduced capital requirements using 10 instead of 5 cohorts
method 1	9.99%	9.99%	9.95%	9.76%	9.25%	19 bp
method 2	9.99%	9.52%	9.33%	9.28%	9.25%	5 bp
method 3	9.99%	9.71%	9.48%	9.37%	9.25%	11 bp
method 4	9.99%	9.60%	9.36%	9.28%	9.25%	8 bp

Table 12: Capital requirements for the weak portfolio using one, two, five, ten, and infinitely many rating cohorts for different methods of defining the sizes of the rating cohorts.

Using more and better dispersed rating classes a bank can save a significant amount of regulatory capital. Depending on the portfolio and on the method of defining the sizes of the rating classes the capital requirements can be reduced by up to 45 bp when using ten instead of five rating classes. On average a bank which increases the number of rating classes from five to ten can expect a lower capital requirement of around 10 to 20 bp. Increasing the number of rating classes from ten to infinity can still be important for certain methods of defining the rating classes. However, for more sophisticated methods (e.g. method 2) the effect is comparably small (around 3 bp). Analysing the results of the capital requirement for the different methods of defining the sizes of rating classes we conclude that method 2 (equal number of customers per rating class) and method 4 (linearly increasing number of expected defaults per rating class) are the most promising concepts. The magnitude of the differences between these two methods is rather small. Method 1 (equally spaced PD intervals) and method 3 (equal number of expected defaults) consistently yielded higher regulatory capital requirements than the others. We explain this outcome by the fact that method 1 and 3 yield comparably broad cohorts for customers with low PDs. Since the relative frequency of low PDs is high for all portfolios under consideration method 1 and 3 produce a rather crude rating system in this setup which implies higher capital requirements due to the concaveity of the Basel risk-weight function.

In order to compare the magnitude of the economic value of reduced capital requirements one has to convert these figures into annual returns by multiplying the reduction in capital requirement with the costs of capital, e.g. a reduction of 20 bp in regulatory capital multiplied by 15% costs of capital translates into a 3 bp increase in the annual return. Compared to the effect of adverse selection the potential of this effect seems to be much lower.

6 Conclusion

In this paper we develop a model to determine the potential economic value of improving a credit rating system. We describe a credit rating system by the number and sizes of the rating classes and the measurement error in the estimation of individual default probabilities. Our model is aimed to advise banks when making an investment decision with respect to the quality of their rating systems. All model parameters are designed to be empirically observable or at least to be calibrated to empirically observable values.

In a first step we analyze the potential effect of adverse selection on the credit portfolio return for rating systems with different accuracy. Our findings indicate that improving a rating system with low accuracy to medium accuracy can increase the annual rate of return by 30 to 40 bp. This effect is even stronger for banks operating in markets with high customer elasticity and high loss rates. Compared to the estimated implementation costs banks could have a strong incentive to invest into their rating system.

Our analysis is restricted to a partial equilibrium framework. Including two or more representative lenders with different rating systems and different implementation costs and modelling the dynamic strategic competition between the lenders would potentially provide more insight to the real-world decision problems banks currently are facing. The framework presented in this paper thus describes the basic decision model that might be used as a starting point in a more sophisticated dynamic setup where modelling the cost differentials of implementing and maintaining rating systems among banks will play a crucial role.

In a second step we analyse the effects of the reduction of regulatory capital requirements under the Basel FIRB approach. Improving the accuracy or rating systems gives the bank the opportunity to make use of a finer grained rating system, i.e. to use a higher number of rating classes. The concaveity of the regulatory capital formula implies that capital requirements are lower for finer grained rating systems. Our results show that the potential of this effect on the portfolio return is rather weak compared to the effect of adverse selection.

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Appendix

Concerning the sizes of the rating classes there is no natural best solution to define PD boundaries. Once the borrowers have been sorted from lowest to highest according to estimated PD, boundary PDs are chosen using one of four methods:

Method 1: The maximal PD for the worst customer is taken and divided by the numbers of cohorts. The resulting value is the stepsize for setting the boundaries.

Example: The maximal PD in a portfolio is 50%. In the case of five cohorts this number is divided by 5 resulting in a stepsize of 10%. So the PD intervals are:
0% - 10%, 10% - 20%, 20% - 30%, 30% - 40%, 40% - 50%.

Method 2: The boundaries are set such that every cohort has the same number of customers.

Example: For 5 cohorts 20% of the customers are in each cohort.

Method 3: The boundaries are set such that every cohort has the same number of defaults.

Example: For 5 cohorts 20% of the defaults are in each cohort.

Method 4: The boundaries are set such that the number of defaults increases linearly from the best to the worst cohort. In this case we have the following relations:

$$\xi \cdot i = A_i \tag{21}$$

$$\sum_{i=1}^{k} A_i = 1 \tag{22}$$

with i index of the cohort

A_i percentage of all defaults in cohort i

k number of cohorts

which are solved for ξ:

$$\xi = \frac{2}{k \cdot (k+1)} \tag{23}$$

Once we know ξ we can use the first equation to calculate how many defaults are in each cohort and with this the boundaries can be calculated.

Example: For five cohorts ξ is equal to:

$$\xi = \frac{2}{5 \cdot (5+1)} = \frac{1}{15} = 0{,}0667$$

Consequently all defaults are distributed over the cohorts as follows:

cohort 1: 6.67% of all defaults are in this cohort

cohort 2: 13.33% of all defaults are in this cohort

cohort 3: 20% of all defaults are in this cohort

cohort 4: 26.67% of all defaults are in this cohort

cohort 5: 33.33% of all defaults are in this cohort